AN ACTIVITY BOOKLET FOR MATHEMATICS LABORATORY

(Higher Secondary Stage)

State Council of Educational Research & Training
Department of School Education, Government of West Bengal
25/3 B.C. Road, Kolkata – 700 019
Email : director@scertwb.org, website : www.scertwb.org

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2020

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Project Director : Dr. Chhanda Ray, Director, SCERT (W.B.)
Project Coordinator : Shri Subrata Kumar Biswas, Research Fellow, SCERT(W.B.)

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Foreward

The State Council of Educational Research and Training, West Bengal (SCERT, W.B.) is a Post Graduate Research and Training Institute under School Education Department, Government of West Bengal. It is the apex body concerning various aspects of School Education. NCF 2005 recommends that Learner's experience in School Education must be linked to the life outside school so that learning experience is joyful and fills the gap between the experiences at home and in community. This Activity Book for Matematics Laboratory will be helpful to understand the concepts of various topics of the Mathematics text books for classes XI and XII. The purpose of this book is not only to convey the process and Philosophy of the laboratory course to Students, Teachers and Teacher Educators but also to provide them appropriate guidances for carrying out activities in the Laboratory and to prepare any Mathematical project. This book is aimed at motivating the reader to design activities on different Mathematical problems. Students, Teachers and Teacher Educators may innovate, modify and improve the activities as per their need. They may adapt or adopt these activities for facilitating the teaching learnig processes. I express my gratitude to all the contrbutors for giving an idea to develop the book.

Finaly I also express my appreciation to our fellow (Mathematics) Shri Subrata Kumar Biswas, MSc, MPhil, B.Ed for his interest in the development of this book. Any constructive comments and suggestions from the readers will be highly appreciated for further improvement of this manual.

Dr. Chhanda Ray
Director
SCERT, West Bengal

July, 2020

List of Contributors

1. Dr Chhanda Ray, Director, State Council of Educational Research & Training, West Bengal

2. Dr. K.A. Sadat, Principal, District Institute of Educational and Training, Uttar Dinajpur.

3. Shri. Suman Sur, Lecturer, District Institute of Educational and Training, North 24 Parganas.

4. Smt. Sudipta Pal, Lecturer, District Institute of Educational and Training, South 24 Parganas.

5. Shri. Subrata Barui, Lecturer, District Institute of Educational and Training, Burdwan.

6. Shri Biplab Mondal, Lecturer, District Institute of Educational and Training, Purulia.

7. Shri. Subrata Kumar Biswas, Research Fellow, State Council of Educational Research & Training, West Bengal

List of Materials

1. Hardboard
2. Coloured pencils
3. Scissors
4. Chart Paper
5. Adhesive
6. Paper (A4)
7. Cardboard
8. Nails
9. Plastic Strip
10. String
11. Match Stick
12. Coloue Ball
13. Wood Board
14. Thermocol Sheet
15. Pin
16. Pencil, Eraser
17. Scale
18. Ruler
19. Wire
20. Eleetric Wire
21. Battery
22. Lamp
23. Transparent Sheet
24. Nylon Wire
25. Hammer
26. Sketch Pen
27. Plywood Board
28. 1V, 2V Bulbs
29. Festul Screws
30. Tester
31. Switch
32. Solderlip Wire
33. Calculator
34. Geoboord
35. Rubber Band
36. Thread
37. Glue
38. Grid Paper
39. Graph Paper
40. Protractor
41. Compus
42. Wire Cutter
43. Metal Dise
44. Marker

CONTENTS

Activity - 1

Objective : To Understand the basic Concept of sets by using Venn-Diagram.

Learning Outcome : Students will be able to the concept of Set theory representation by Venn Diagram.

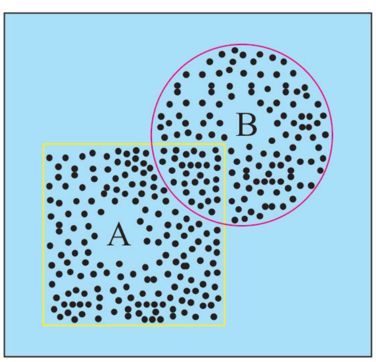
Materials : Hardboard, different Coloured pencils, Scissors, Chart Paper, Adhesive, white sheets of Paper.

Preparation :

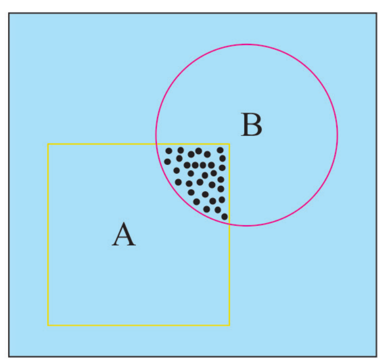
1. Take a rectangular shape Hardboard and Cover it with chart paper.
2. Take a rectangular shape A on Hardboard and take a circle shape B as shown in the figure.

Demonstration :

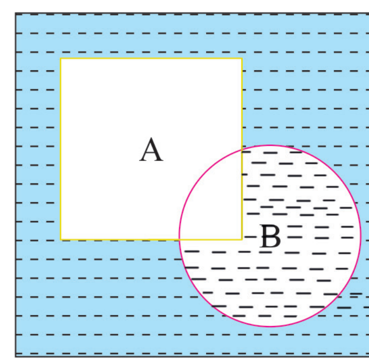
1. U denotes the Universal set represented by the rectangle.
2. Square A and Circle B represent the subset of the Universal Set U as shown in the figure.
3. A′ denote the Complement of the set A and B′ denote the complement of the set B as shown in the figure.
4. Coloured portion represent $A \cup B$, $A \cap B$, A', B' $(A \cup B)'$, $(A \cap B)'$.



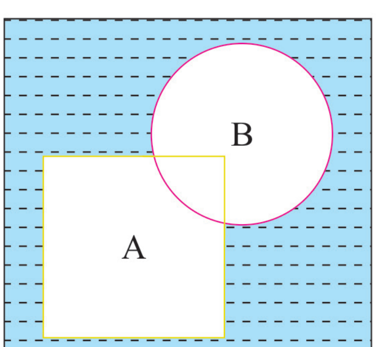
$A \cup B$



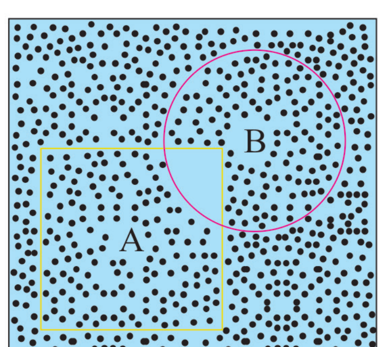
$A \cap B$



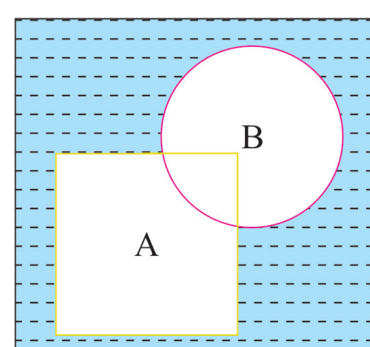
A'



B'



$(A \cap B)'$



$(A \cup B)'$

Activity - 2

Objective : To find the subsets of a given set and verify that if a set has n elements then the total member of subsets is 2^n.

Learning Outcome :

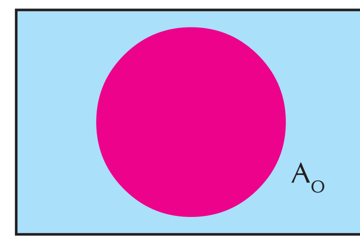
From this activity student will be able to Calculate the number of subsets of a given set.

Materials : Hardboard, different Coloured pencils paper.

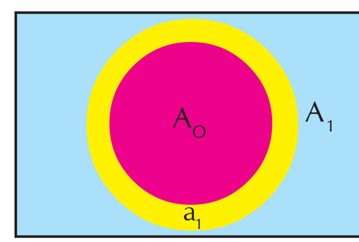
Preparation :
1. Take the empty set (A_O) which has no element.
2. Take three sets A_1, A_2, & A_3 which have one, two & three elements respectively.

Demonstration :

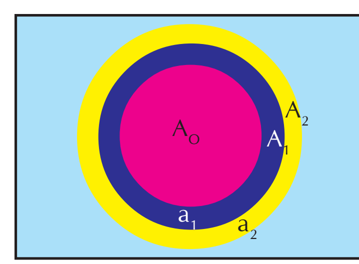
1. Represent A_O,
   Possible subset of A_O
   is A_O. (denoted by p).
   The number of Subsets of A_0 is $1 = 2^{0.}$



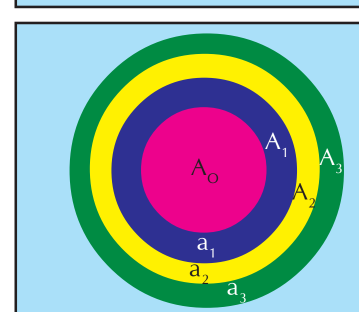
2. Represent A_1,
   Subsets of A_1 are
   ϕ, $\{a_1\}$.
   The number of subsets of A_1 is $2 = 2^1$.



3. Represent A_2,
   Subsets of A_2 are ϕ_1,
   $\{a_1\}$, $\{a_2\}$, $\{a_1, a_2\}$.
   The number of subsets of A_2 is $4 = 2^2$.



4. Represent A_3
   Subset of A_3 are $\{a_1\}$, $\{a_2\}$, $\{a_3\}$, $\{a_1, a_2\}$, $\{a_1, a_3\}$, $\{a_2, a_3\}$ $\{a_1, a_2, a_3\}$.
   The number of Subsets of A_3 is $= 8 = 2^3$.



Continuing this way, the number of subsets of Set A containing n elements $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_n$ is 2^n.

Activity - 3

Objective : To Verify distributive law for three given non.empty sets A, B & C. that is

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Learning Outcome :

Students will be able to learn various laws of sets.

Materials : Hardboard, different Coloured pencils Scissors, adhesive.

Preparation :
1. Cut five rectangular shape from a Hardboard.
2. Draw three circles and mark them as A, B & C in each of the five rectangles as shown in the figures.

Demonstration :

1. U Denotes the Universal Set represented by the rectangle in each figure.
2. Circles A, B & C represent the subsets of the Universal Set U.
3. Coloured Portion represente $B \cap C$ (fig:1).

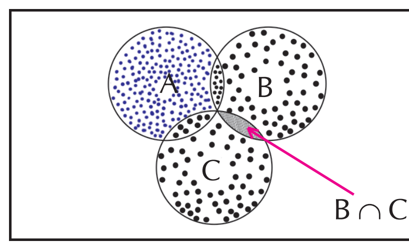


fig:1

4. Coloured Portion represents $A \cup B$ (fig:2).

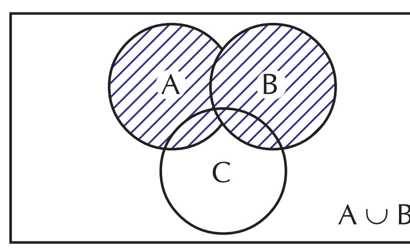


fig:2

5. Coloured portion represents $A \cup C$ (fig:3).

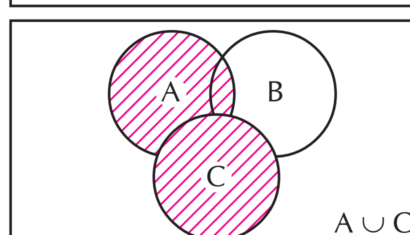


fig:3

Thus the distributive law is verified by Venn Diagram (fig:4)

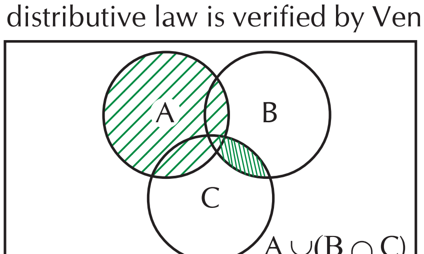


fig:4

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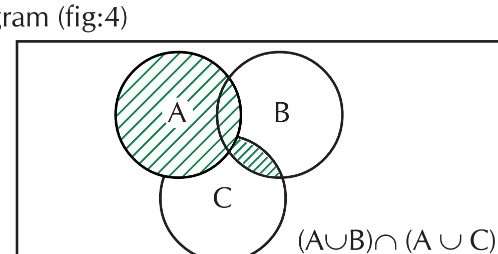


fig:5

Activity - 4

Objective : To demonstrate a function which is **one-one.**

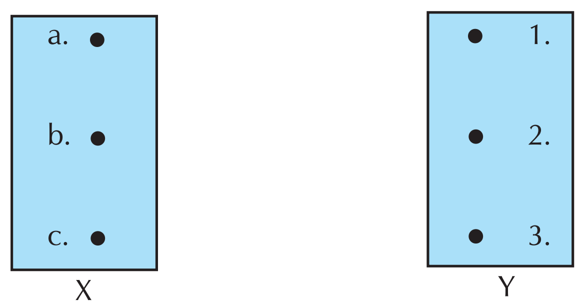
Learning Outcome :

Learners will be able to understand the concept of one-one function.

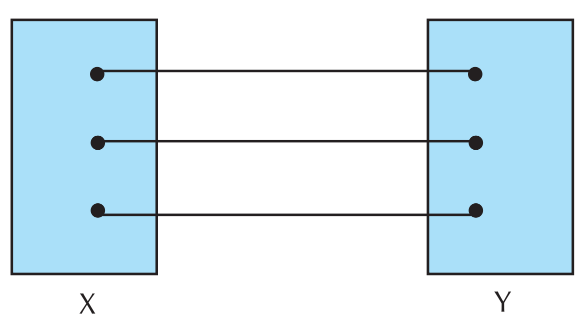
Materials : Cardboard, nails, strings, adhesive and Plastic strips.

Preparation :

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails in it as shown in the figure as a, b, c.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the figure name as 1, 2 & 3.



3. Join nails on the left strip to the nails on the right strip as shown in the figure.



Demonstration :

1. Take the set X = {a, b, c}
2. Take the set Y = {1, 2, 3}
3. Join the elements of X to the elements of as shown in the figure.
4. The image of the elements a, b & c of X in Y are 1, 2, 3 respectively.
5. Every Element in X has a image in Y. So, the function is one-one.

Activity - 5

Objective : To demonstrate a function which is **onto.**

Learning Outcome :

Learners will be able to understand the concept of onto function.

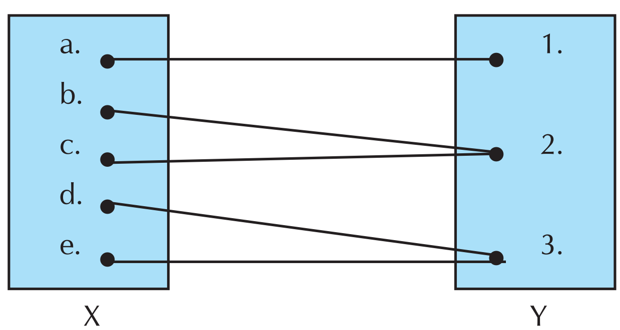
Materials : Cardboard, nails, strings, adhesive and plastic strips.

Preparation :

1. Paste a plastic strip on the left hand side of the cardboard and fix five nails in it as shown in the figure as a, b, c, d, e.
2. Paste a plastic strip on the right hand side of the cardboard and fix three nails in it. as shown in the figure name as 1, 2, 3.

Demonstration :

3. Join nails on the left strip to the nails on the right strip as shown in the figure.



Demonstration :

1. Take the set Y = {1, 2, 3}
2. Take the set X = {a, b, c, d, e}
3. Join (Correspondence) element of X to the elements of Y as shown in the figure.
4. The pre-image of each element of Y in X exists. So, the function is onto.

Activity - 6

Objective : To obtain formula for the sum of odd number of first natural n numbers, by mathematical Induction.

Learning Outcome :

Learners will be able to find sum of odd numbers of first n natural numbers.

Materials : Different coloured plastic unit cubes, Hardboard, adhesive.

Preparation : 1. Take 1 $(=1)^2$ plastic unit cube.

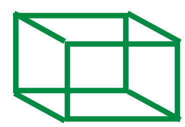


fig:1

2. Take $4(=2)^2$ plastic unit cubes and form a cuboid as shown in figure.

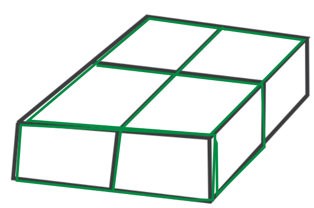


fig:2

3. Take a $(=3)^2$ plastic unit cubes and form a cuboid as shown in the figure. $1+3=4$

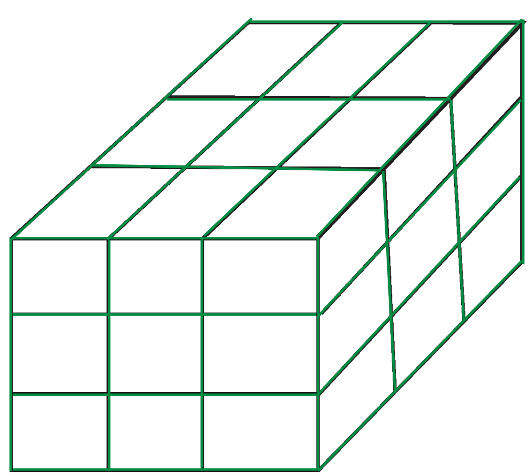


fig:3

4. Take $16(=4)^2$ plastic unit cubes and form a cuboid as shown in the figure and so on.
$1+3+5+7$

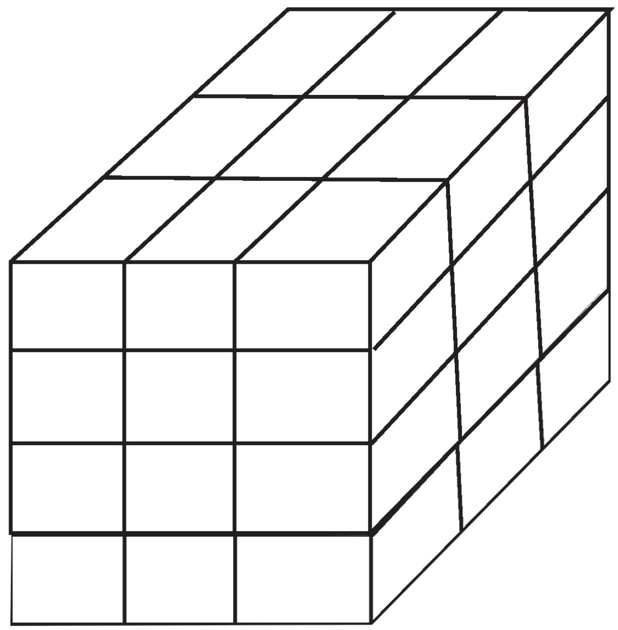


fig:4

Demonstration : 1. Area of one set as given in fig. 1

$1^2 = 1$

2. Area of one set as given in fig. 2

$2^2 = 4 = 1+3$

3. Area of one set as given in fig. 3

$3^2 = 9 = 1+3+5$

4. Area of one set as given in fig. 4

$4^2 = 16 = 1+3+5+7$

$\therefore\ 1+3+5+7 = (4)^2 . \Rightarrow\ 1+3+5+\{2.4.-1\} = (4)^2.$

Assume this is true for $n = k$.

$$1+3+5+\ldots\ldots\ldots\ldots\ldots\ldots + (2k-1) = k^2$$

$\therefore$ Now for $n = k + 1$,

$1+3+5+\ldots\ldots\ldots+(2k+1) + \{2(k+1)-1\}$

$= k^2 + 2k+2-1$

$= k^2 + 2k + 1$

$= (k+1)^2$

Hence, we prove sum of odd number of first n-natural numbers by Mathematical Induction.

N.B. –The first tile falls and in the event that any tile falls its successor necessarily falls.

Activity - 7

Objective : To construct Pascal's triangle and write binomial expressions.

Learning Outcome :

Learners will be able to understand binomoal theorem.

Materials Required : One Blank Sheet, Matchsticks

Preparation :

(i) Place a blank sheet on the table.

(ii) Join three Matchsticks to construct a triangle. fig. 1
[Numbering the two open end as 1 & 1].

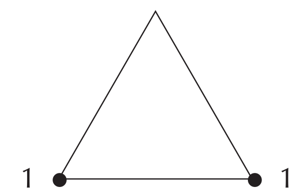


Figure : 1

(iii) Construct same triangle as stated above with the two horizontal end of the 1st triangle. Numbering the open ends as shown in the fig. 2.

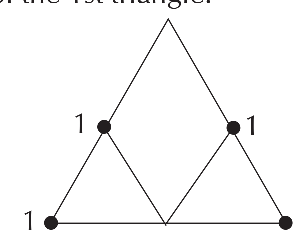


Figure : 2

(iv) Continue this process for 7 to 8 steps as shown in the fig. 3

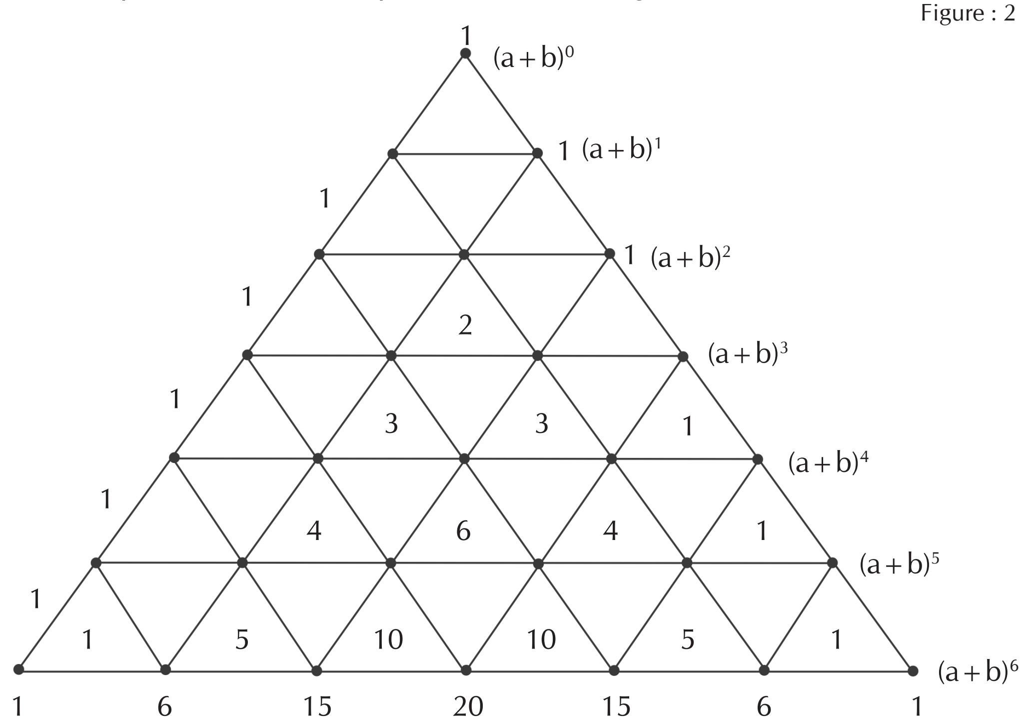


Figure : 3

Demonstration :

(1) For the 1st horizontal line of the 1st triangle, we numbering the two end as 1 & 1 respectively. It will expand $(a+b)^1$. i.e. $(a + b)^1 = a + b$.

(2) Now in the 3rd row start & end with 1 and complete interior term by summing the two numbers above it. i.e. [1,2,1].Use these values as the Coefficient of expansion of $(a + b)^2$.

(3) On each subsequent row start & end with 1's and compute each interior term by summing the two numbers above it.

Now in this way we can expand binomials with the help of the values of the row of Pascal's triangle as the Coefficient of the expansion.

For Example : Let's expand $(a + b)^5$

Since we are raising (a + b) to 5th Power, use the values of the 6th row of Pascal's triangle as the Coefficient of your expansion.

i.e., $(a + b)^5 = 1.\,a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1.b^5$

Activity - 8

Objective : To develop the concept of Permutation.

Learning Outcome :

After this Activity the learner will be able to understand the concept of permutation.

Materials Required : Coloured balls

Preparation :

(i) First take one blue ball. Now it may be arranged in only one way. So No of Permutation $= 1$

(ii) Next take one blue ball & one red ball, Now these two balls may be arranged in 2×1 different ways. So for this case number of Permutation will be $= 2 \times 1 = 2$

(iii) Take three balls of different colour. Now these three balls may be arranged in $3 \times 2 \times 1$ different ways. So number of Permutation will be 6.

(iv) Continue this Steps 6 to 7 times.

Ball	No. of Permutation
	$1 = 1!$
	$2 \times 1 = 2 = 2!$
	$3 \times 2 \times 1 = 6 = 3!$
	$4 \times 3 \times 2 \times 1 = 24 = 4!$
	$5 \times 4 \times 3 \times 2 \times 1 = 120 = 5!$
	$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 = 6!$

Activity - 9

Objective : To understand $^np_r = \frac{n!}{(n-r)!}$

Learning Outcomes :

i. Learners will be able to varify the formula ^np_r

ii. Students will be able to apply the concept of permutation in communication networks.

Materials Required : Coloured balls, wood board

Preparation : Take a wood board & make holes on it. (As per requirement).

Demonstration :

(i) 1st take 2 different coloured balls and arrange them in one hole. You see that you can arrange the balls in two different ways.

$$\frac{\text{2 different balls}}{\text{1 hole}} \qquad \text{No. of arrangement} = 2 \text{ i.e. } \frac{2\times 1}{1}$$

$$\Rightarrow \frac{2!}{1!} \Rightarrow \frac{2!}{(2-1)!}$$

(ii) Now take 3 different coloured balls and arrange them in 1 hole. Here you can arrange them in 3 different way.

$$\text{i.e. } \frac{\text{3 different balls}}{\text{1 hole}} \qquad \text{No. of arrangement} = 3 \Rightarrow \frac{3\times 2\times 1}{2\times 1}$$

$$\text{i.e. } \frac{3!}{2!}$$

$$\text{i.e. } \frac{3!}{(3-1)!}$$

(iii) Next take 3 different coloured balls & arrange them in 2 hole. Here you can arrange them in 6 different ways.

i.e $\frac{\text{3different balls}}{\text{1st hole}}$ $\frac{\text{2different balls}}{\text{2nd hole}}$ No. of arrangement $6 = 3 \times 2$, it can be written as $\frac{3 \times 2 \times 1}{1}$

i.e. $\frac{3!}{1!}$

$= \frac{3!}{(3-2)!}$

(iv) Take 4 balls & arrange them in 2 holes. Here you can arrange them in

$\frac{\text{4different balls}}{\text{1st hole}}$ $\frac{\text{3different balls}}{\text{2nd hole}}$ No. of arrangement $= 12 = 4 \times 3$

i.e $\frac{4 \times 3 \times 2 \times 1}{2 \times 1}$

$= \frac{4!}{2!}$

$= \frac{4!}{(4-2)!}$

(v) Similarly take 5 balls & arrange them in 3 holes. Here you can arrange them in 60 different ways.

$\frac{\text{5 different ball}}{\text{1st hole}}$ $\frac{\text{4 different ball}}{\text{2nd hole}}$ $\frac{\text{3 different ball}}{\text{3rd hole}}$

$\therefore$ No of arrangement $= 5 \times 4 \times 3$

i.e, $\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$

i.e, $\frac{5!}{2!}$

i.e, $\frac{5!}{(3-2)!}$

Now, from the above steps we can write that,

No. of permutation $= \frac{n!}{(n-r)!}$

Where we have to arrange *n* different objects in *r* places ($r \leq n$)

Activity - 10

Objective : To Prove that $1+2+3+\dots\dots\dots\dots+n = \frac{n(n+1)}{2}$

Learning Outcome :

(i) Students will be able to find the sum of n natural numbers.

Materials Required : Card board, Colour, Scissors

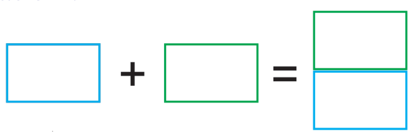
Preparation : (i) Cut the Card board into unit Squares.

(ii) Colour the unit squares.

(iii) Place them as demonstrated below.

Demonstration :

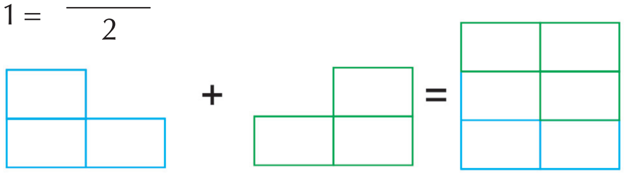
Step - 1 :



Area 1(1 + 1)

$\therefore \quad 1 = \frac{1(1+1)}{2}$

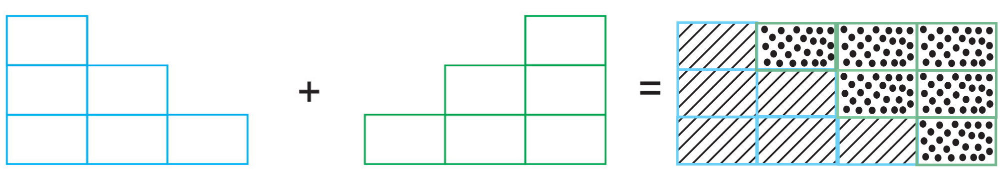
Step - 2 :



Area = 2(2 + 1)

$\therefore \quad 1 + 2 = \frac{2(2+1)}{2}$

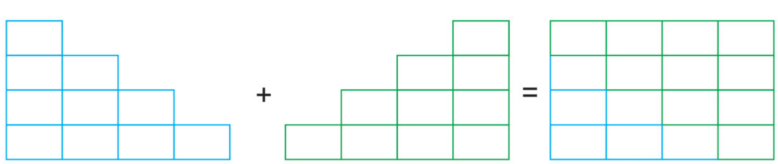
Step - 3 :



Area = 3(3 + 1)

$\therefore \quad 1 + 2 + 3 = \frac{3(3+1)}{2}$

Step - 4



Area = 4(4 + 1)

$\therefore \quad 1 + 2 + 3 + 4 = \frac{4(4+1)}{2}$ and so on

So, $1+2+3+4+\dots\dots\dots\dots\dots\dots+n = \frac{n(n+1)}{2}$

Activity - 11

Objective : To establish AM $\geq$ G.M

Learning Outcome :

After complete this activity the learner will be able to understand the concept of A.M & G.M.

Materials Required : 4 pieces of Card board of area ab.

Preparation : Arrange the 4 pieces of Card board of area ab as a square of side $(a+b)$.

Demonstration :

Area of square ABCD = Area of the square PQRS + 4 $\times$ area of the rectangle of side a & b.

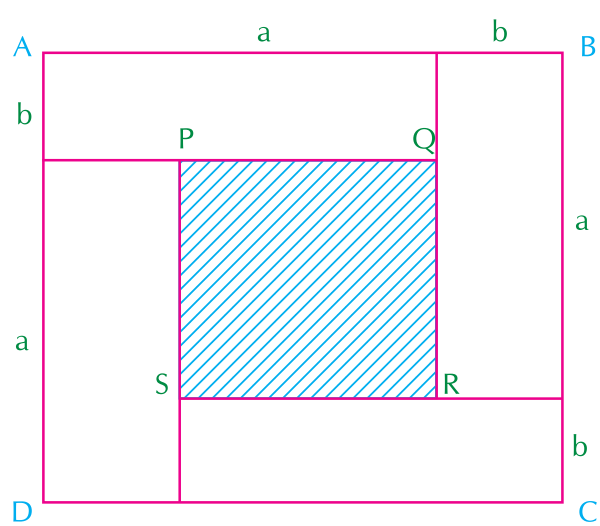
$$(a+b)^2 = (a-b)^2 + 4ab$$

or, $(a+b)^2 \geq 4ab$ [equality holds only when $a=b$]

or, $\left(\frac{a+b}{2}\right)^2 \geq ab$

or, $\frac{a+b}{2} \geq \sqrt{ab}$

$\therefore$ A.M $\geq$ G.M



Activity - 12

Objective : To understand the concept of number series and its sum.

Learning Outcome :

From the Model students will be able to demonstrate the following :

1. Concept of number series that is arithmatic Progression.
2. Sum of number series.

Materials Required : Plastic strip, Chart Papers, thermocol sheets, adhesive.

Preperation :

1. Take a thermocol sheet in the shape of the rectangle ABCD.
2. Take some plastic strips each of equal fixed length denoted by 'a' and some plastic strips, each of equal fixed length denoted by 'd'
3. Arrange and paste both types of strips so as to get terms a, a+d, a+2d, ..., a+9d placed at unit distance apart and arrange along the rectangle as shown in the fig 1.
4. The last strip ends in F along BC, extend F to C by a fixed length a so as to cut rectangle ABCD.

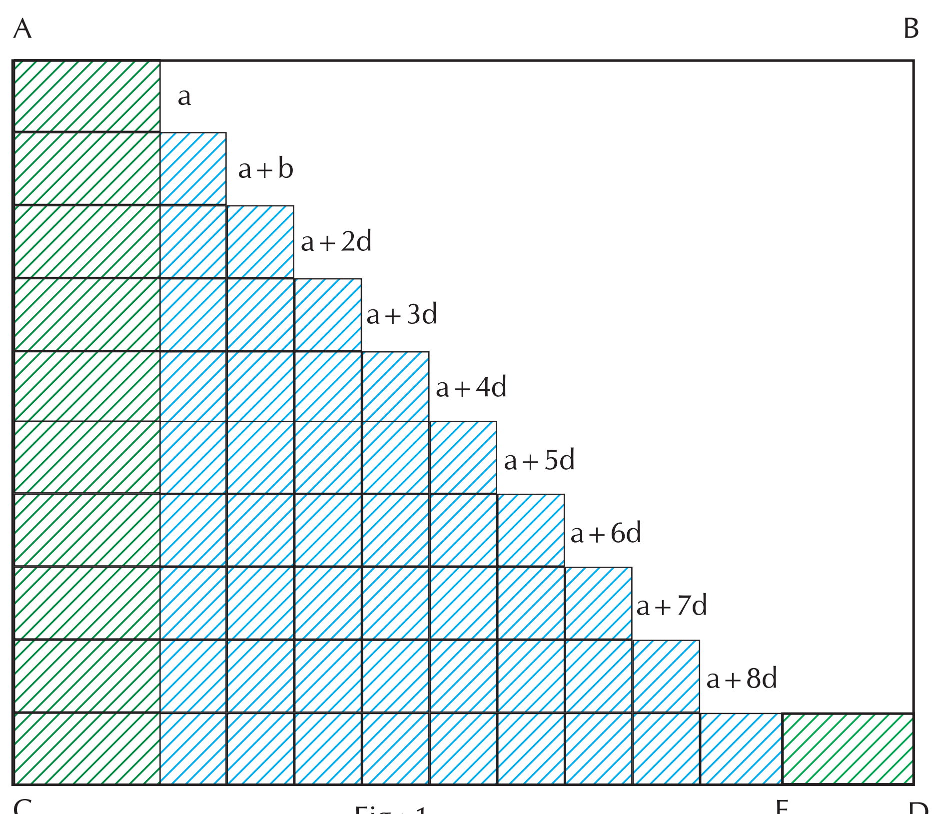


Fig : 1

Demonstration :

1. The first strip is of length a
2. The second strip is of length $a+d$
3. The third strip is of length $a+2d$
4. The fourth strip is of length $a + 3d$
5. The fifth strip is of length $a + 4d$
6. The sixth strip is of length $a + 5d$
7. The seventh strip is of length $a + 6d$
8. The eighth strip is of length $a + 7d$
9. The ninth strip is of length $a + 8d$
10. The tenth strip is of length $2a + 9d$

Observation :

1. All the strips arranged look like a staircase.
2. All the numbers having a common difference i.e. d. So numbers are in arithmetic progression :
3. The sum of the above number series

$= a + (a + d) + (a + 2d) + \dots\dots\dots + (a + 9d)$

$= 10a + 45d = 5(2a + 9d)$

$= \frac{10}{2}(2a + 9d) = \frac{1}{2}$ (Area of rectangle ABCD whose length BC $= 2a+9b$ breadth is 10 units)

4. The sum $\frac{10}{2}[2a + 9d] = \frac{10}{2}[a + a + 9d]$

$= \frac{\text{Number of terms}}{2}[\text{1st term} + \text{Last term}]$

5. Last term $= a + 9d =$ 1st term + (Number of terms–1) $\times$ Common difference

Note : This model can be prepared by wooden board for Laboratory.

Activity - 13

Objective : To find the sum of odd numbers.

Learning Outcome :

From the Model student will be able to explain the sum of odd numbers by taking different values of n.

Materials Required : Thermocol Sheet, thermocol balls, pins, pencil, scale, adhesive, chart paper.

Preparation :

1. Take a square piece of the thermocol.
2. Fix chart paper on the thermocol.
3. Draw horizontal and vertical lines with pencil to make squares.
4. Take a pin and fix a thermocol ball in it and fix it in the corner of the square.
5. Repeat the same with thermocol balls and pins on the whole board as shown in the fig : 1.

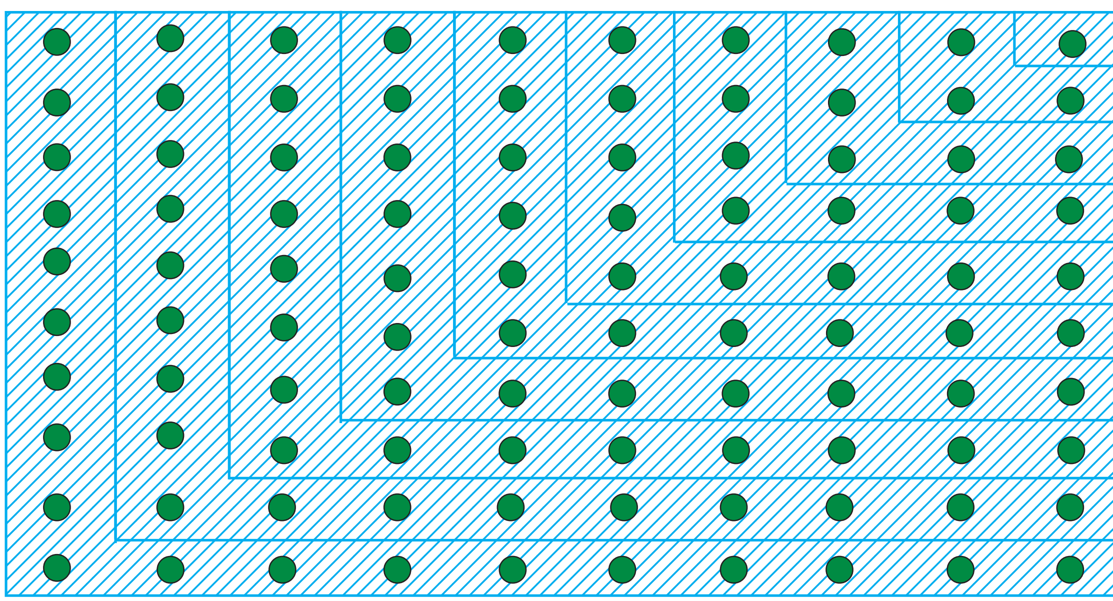


Fig : 1

Demonstration :

1. For n = 1, the square on the top of the right corner, in the figure having one ball. it can be represent as 1^2
2. For $n = 2 \Rightarrow \sum(2n-1) = (2\times2-1) + (2\times1-1) = 3 + 1 = 2^2$
   i.e. the next square having 1 + 3 = 4 balls can be represent as 2^2
3. The next square having 1 + 3 + 5 = 9 balls can be represent as 3^2
4. The next square having 1 + 3 + 5 + 7 = 16 balls can be represent as 4^2
5. Taking the sum of all the cases, we get $1 + 3 + 5 + 7 + \ldots.. + (2n-1) = n^2$

Note : The model can also be prepared from wooden board and nails for Laboratory.

Activity - 14

Objective : To understand domain of solutions to an inequation on a number line.

Learning Outcome :

After this activity the learner will be able to understand domain of solutions to an inequation on a number line.

Materials Required : Cardboard, Thick white Paper, Sketch Pen, Ruler, Wire, Adhesive, Scissors etc.

Method of construction :

1. Take a Cardboard of a convenient Size & Paste a white paper on it.
2. Draw a line x^1ox to represent X-axis.
3. Mark the line x^1ox with the help of ruler & Sketch Pen as shown in Fig. - 1.

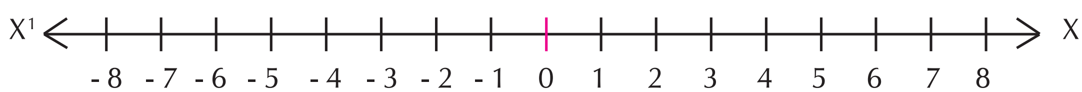


Fig. - 1

4. Consider the linear in-equation of one degree with one variable viz -
   a) $x + 5 \leq 9$
   b) $7x < -21$
   c) $5x - 6 \geq -16$
   d) $-9 < 3x \leq 15$
5. Draw the inequation $x + 5 \leq 9$ or, $x \leq 4$ on the number line as shown in Fig.-2.

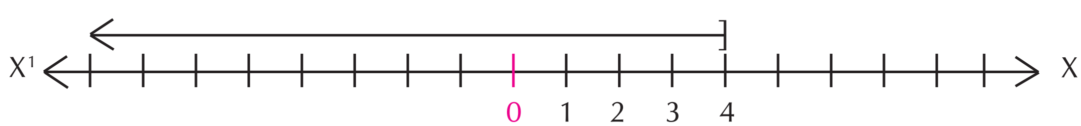


Fig. - 2

Symbols '[,]' represents $\geq$ & $\leq$ respectively.

6. Draw the inequation $-9 < 3x \leq 15$ or, $-3 < x \leq 5$ on the number line as shown in Fig.-3.

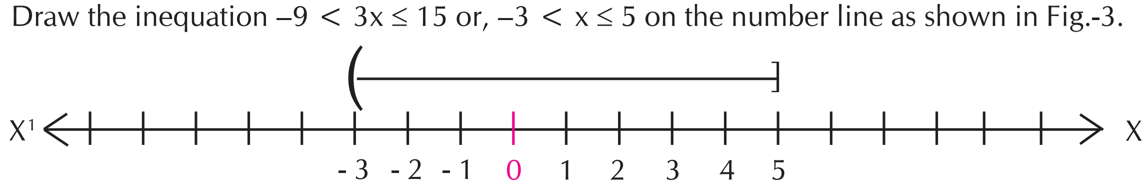


Fig. - 3

Symbols '(,)' represents $<$ & $>$ respectively.

'[,]' represents $\geq$ & $\leq$ respectively.

7. Consider the linear inequation of two degree with one variable viz -
   a) $(x+1)(x-1) \geq o, x \in R$
   b) $(x+1)(x-1) < o, x \in R$
   c) $(x+3)(x-3) > o, x \in R$
   d) $(x+3)(x-3) \leq o, x \in R$
8. Draw the inequation $(x+1)(x-1) \geq o$ on the number line with the help of wire as shown in Fig-4.

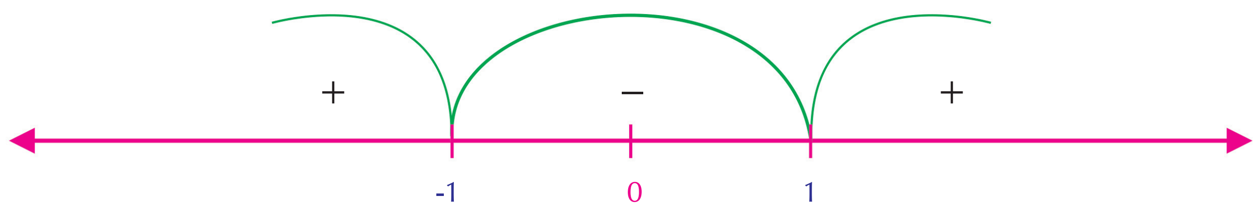


Fig-4

9. Consider the linear in equation of three degree with one variable viz -

   $(x-1)(x+1)(x-2) < o, x \in IR$.
10. Draw the inequation $(x-1)(x+1)(x-2) \leq o$ on the number line with the help of as wire shown in Fig.-5.

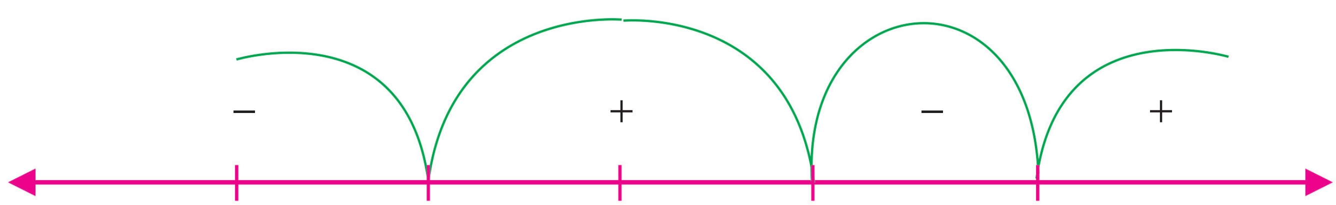


Fig.-5

Demonstration :

1. Mark the point O as origin.
2. In figure 2, inequation $x+5 \leq 9$ or, $x \leq 4$ gives us the domain $(-\infty, 4]$.
3. In figure 3, inequation $-9 < 3x \leq 15$ or, $-3 < x \leq 5$ gives us the domain $(-3, 5]$.
4. In figure 4, inequation $(x+1)(x-1)$ o gives us the domain $[-\infty, -1] \cup [-\infty, -1]$.
5. In figure 5, inequation $(x-1)(x+1(x-2) < 0$ gives us the domain $[-1, 2)$.

Activity - 15

Objective : **To understand the application of switch connection in parallel with Mathematical Logic.**

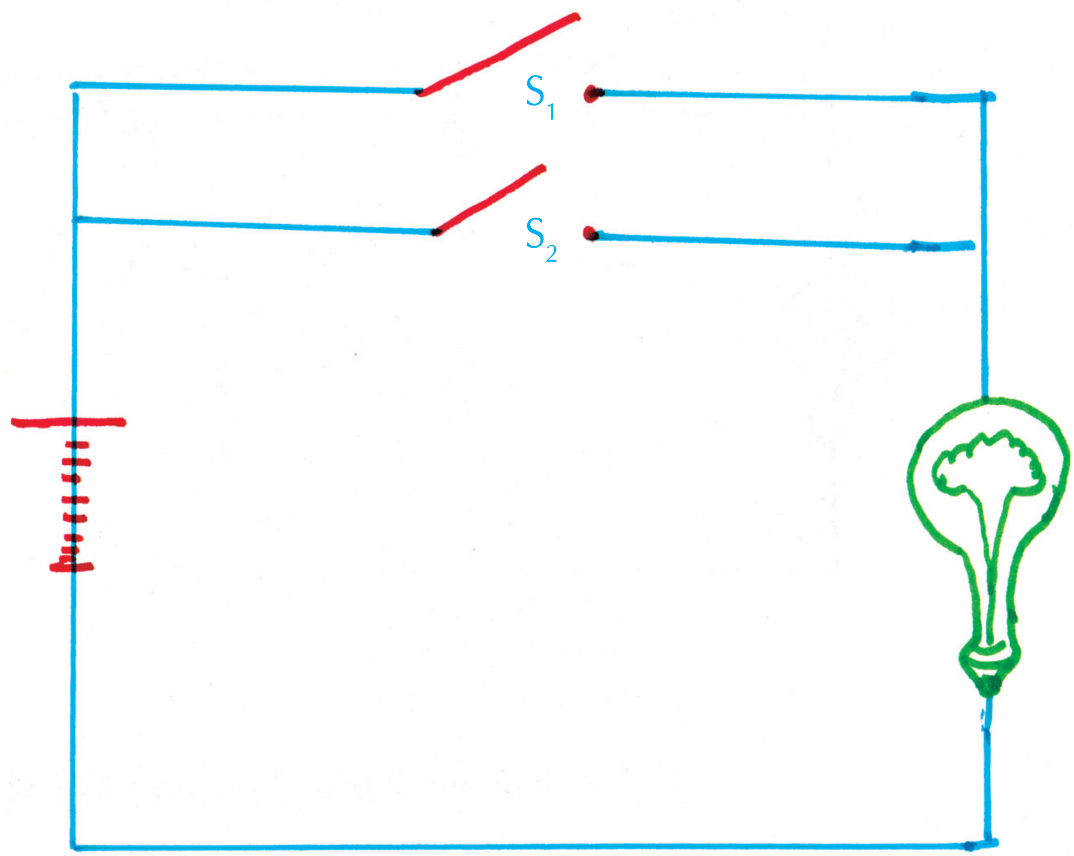
Learning Outcome :

1. For this activity the students will be able to learn mathematical reasoning.
2. The students will be able to explain application of switch connection in parallel.

Materials Required : **Switches S_1, S_2, Electric Wire, Battery and Lamp.**

Preparation :

1. Connect switches S_1, S_2 in Parallal.
2. Connect battery and Lamp as shown in the figure.



Figure

Demonstration :

1. Lamp will glow if any of the switches closed.
2. Lamp will glow if both the switches closed.
3. Lamp will not glow of both the switches open.

4. Switches closed means 1 and switches open means zero.

5. Assign, statements p, ~p; q, ~q to the status of switches S_1 closed, open and S_2 closed, open.

Observation :

1. p v q = 1, ~p v q = 1; p v ~p = 1, ~ p v ~q = o

2.

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

Activity - 16

Objective : To understand the application of series connection of switches to Mathematical logic.

Learning Outcome :

1. From this activity students will be able to explain application of switch connection in series.
2. For this activity the students will be able to understand Mathematical reasoning.

Materials Required : Switehes, Electric Wire, Battery and Lamp.

Preparation :

1. Take two switches S_1 and S_2 in the series as shown in the figure -1.
2. Connect battery and lamp so as to complete the circuit.

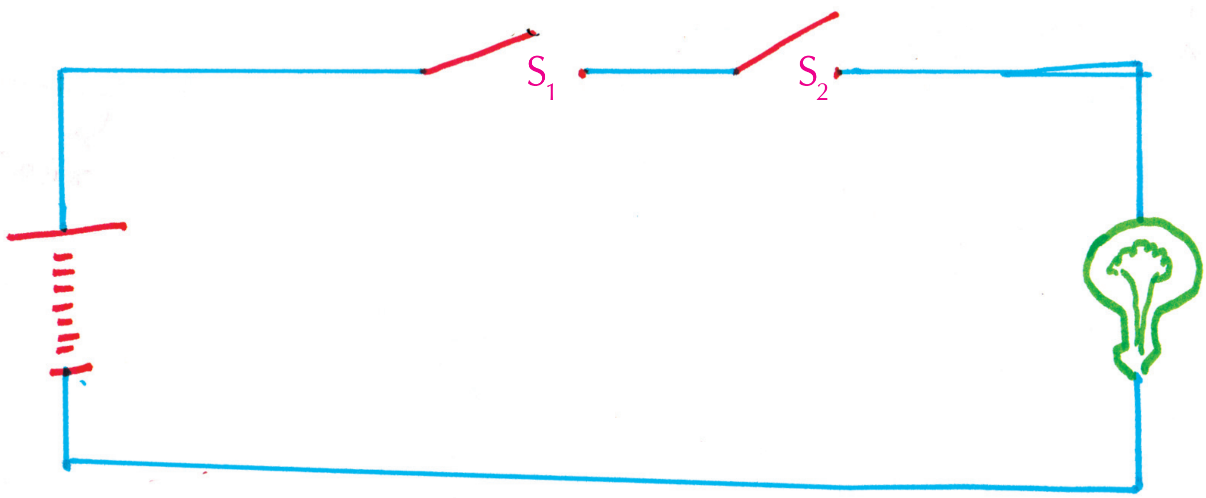


Figure-1

Demonstration :

1. Assign statement p, ~ p; q, ~q to the status of switches S_1 and S_2 respectively.
2. p represent switch S_1 is closed $\equiv 1$

   ~ p represent switch S_1 is open $\equiv 0$

   q represent switch S_2 is closed $\equiv 1$

   ~ q represent switch S_2 is open $\equiv 0$

3. For demonstrating status of switches and lamp's on a off position the following table may be used.

status of switches S_1 S_2	Denotes series switch operation	Status of Lamp
0 1	0^0	0 → Lamp is off
0 1	0^1	0 → Lamp is off
1 0	1^0	0 → Lamp is off
1 1	1^1	1 → Lamp is off

Observation :

1. $p \wedge q = 1, \sim p \wedge q = 0; p \wedge \sim q = 0, \sim p \wedge \sim q = 0$

2.

p	q	p^q
T	T	T
T	F	F
F	T	F
F	F	F

This is truth table

Activity - 17

Objective : To understand Conic section in 2D.

Learning Outcome :

Learners will be able to understant the conic sections in 2D.

Materials Required : Trasparent sheet, Scissors, hard board, adhesive

Preparation :

1. Cut a trasparent sheet in the shape of sector of a circle.
2. Form a right circulur cone by folding the trasparent sheet and using adhesive.
3. Fix the cone on the hard board.
4. Cut through a cone at different angles to the base.

Demonstration :

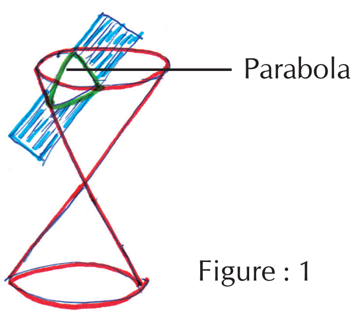


Figure : 1

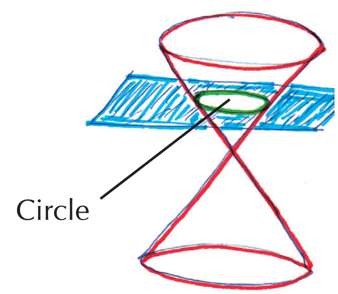


Figure : 2

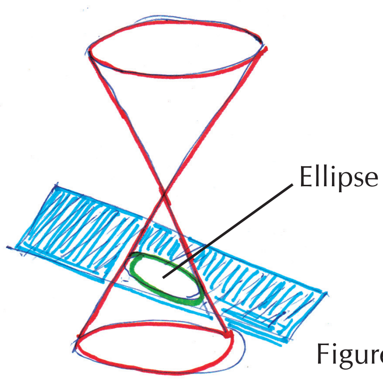


Figure : 3

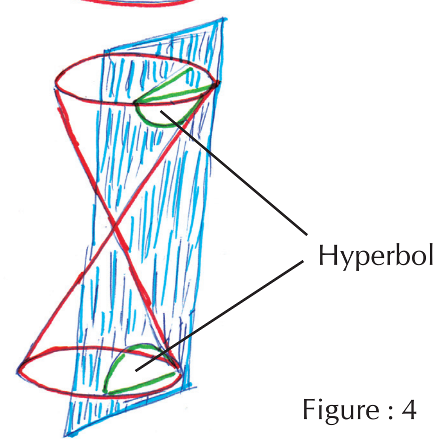


Figure : 4

1. Place the plane such that it is parallel to a generator. The section will be a parabola. (Fig. 1)
2. Place the Plane in such a way that it is perpendicular to the axis of the cone then the section will be circle (Fig. 2)
3. Place the Plane such that it is inclined slighthy to axis. The section will be an ellipse (Fig. 3)
4. Take a double right circular cone and place the plane in such a way that it is parallel to the axis and cuts upper parts as well as the lower part of the double cone. The sution will be hyperbola. (Fig. 4)

Activity - 18

Objective : To construct a parabola when distance between Directix and focus is given.

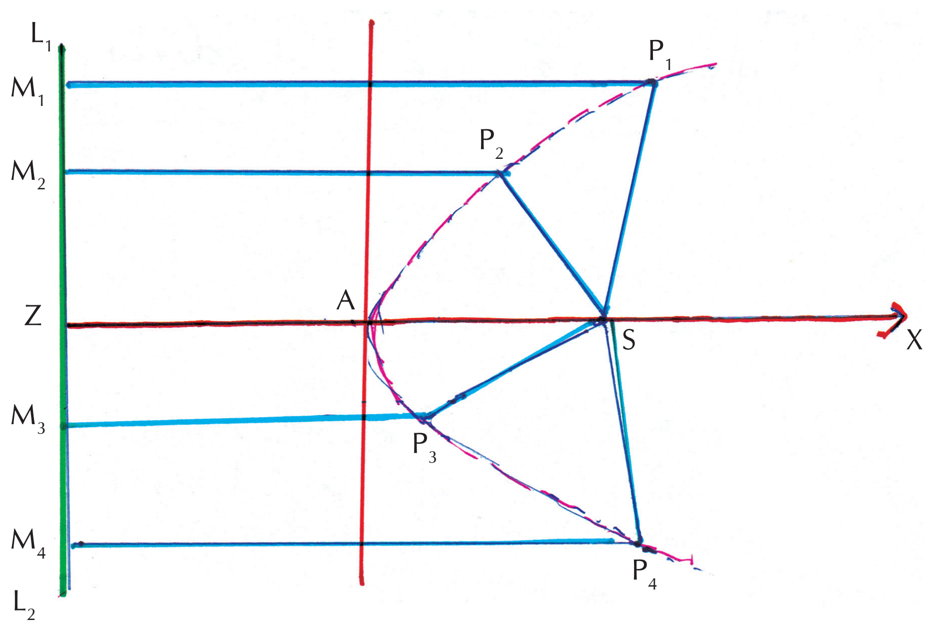
Learning Outcomes :

1. From this students will be able to construct a parabola when focus and directrix are given.

Materials Required : Hard board, Chart paper, hammer, nails nylon wire/string.

Preparation :

1. Take a rectangular hard board and cover it with chart paper.
2. Draw ZX horizontally as axis on the chart paper and mark focus S on ZX. Also draw a vertical line L_1L_2 through Z to denote directrix.
3. Bisect ZS in A to denote the vertex.
4. Take a ratio $\overline{SP_1}$ and $\overline{P_1M_1}$, $\overline{SP_2}$ and $\overline{P_2M_2}$, $\overline{SP_3}$ and $\overline{P_3M_3}$, $\overline{SP_4}$ and $\overline{P_4M_4}$ etc as shown in the figure.



Figure

5. Fix nails on these points P_1, P_2, P_3, P_4, M_1, M_2, M_3, M_4 etc. and join by wire $\overparen{P_1P_2P_3P_4}$ to get the shape of parabola.

Demonstration :

1. ZX denotes axis.
2. L_1L_2 denotes directrix.

3. $\frac{SP_1}{P_1M_1} = \frac{SP_2}{P_2M_2} = \frac{SP_3}{P_3M_3} = \frac{SP_4}{P_4M_4}$

4. Locus of P (i.e. $\overset{\frown}{P_1P_2P_3P_4}$...) is a parabola.

Observation :

1. Here SP = PM, as all the ratio and equal to 1.
2. Fixed point S called focus.
3. A denotes Vertex.
4. $\overline{AS} = \overline{AZ}$ ie A divides ZS equally.

Activity - 19

Objective : To draw an ellipse in 2D.

Learning Outcome :

Students will be able to draw an ellipse.

Materials Required : Rectangular Card board/hard board, coloured chart paper, sketch pen, scale, adhesive.

Preparation :

1. Take a rectangular hard board and cover it with chart paper.
2. Take a rectangle ABCD as shown in the figure and divide to B_1B_2 and draw horizontal line A_1A_2 as axis on the chart paper.
3. Mark S_1 & S_2 on A_1A_2 as focii.

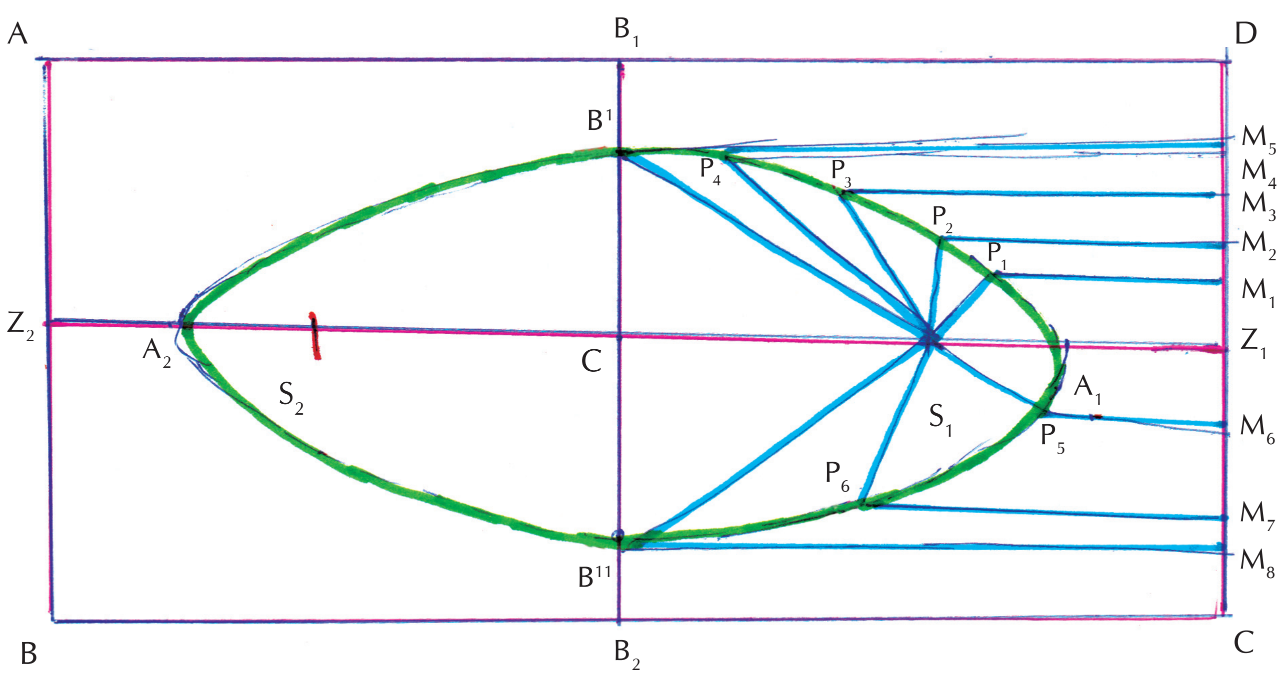


Figure-1

4. Take a ratio $\overline{S_1P_1}:\overline{P_1M_1}$; $\overline{S_1P_2}:\overline{P_2M_2}$; $\overline{S_1P_3}:\overline{P_3M_3}$; $\overline{S_1P_4}:\overline{P_4M_4}$; $\overline{S_1B'}:\overline{B'M_5}$; $\overline{S_1P_5}:\overline{P_5M_6}$; $\overline{S_1P_6}:\overline{P_6M_7}$; $\overline{S_1B''}:\overline{B''M_8}$ as shown in the figure-1.
5. Fix nails on these points as join by wire to make part of an ellipse.
6. Repeat this process in the other part of the rectangle.

Demonstration :

1. $\overleftrightarrow{B_1B_2}$ and $\overleftrightarrow{Z_1Z_2}$ represent axis (one is called Major another one is Minor)
2. Intersecting point C is called centre of the ellipse.
3. S_1, S_2 are the focii.
4. A_1, A_2 reprerents Vertics and B', B'' are also the vertices.

Observation :

1. The ratios are constant < 1.
2. It is a central conic.

Activity - 20

Objective : **To visualise the position and co-ordinates of a point in 3D.**

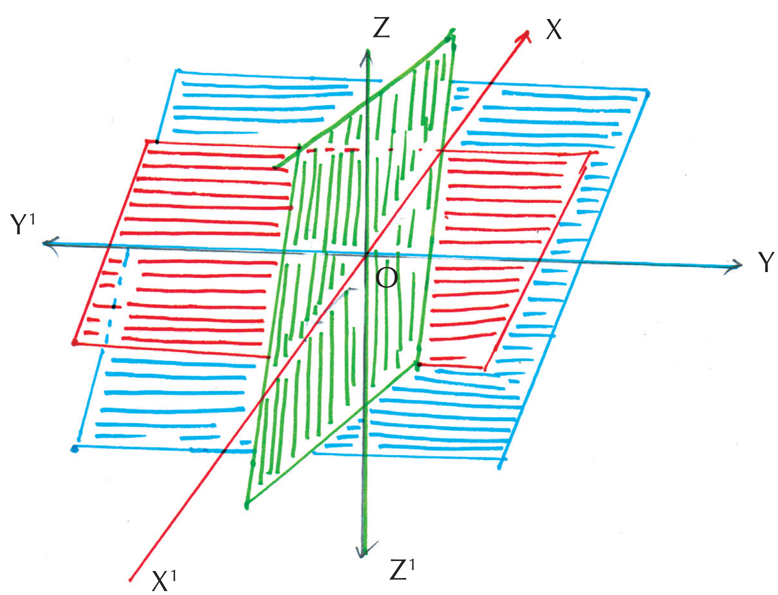
Learning Outcome :

Students will be able to visualise the position and co-ordinates of a point in 3D space.

Materials Required : **Plywood board, wires, nails.**

Preparation :

1. Take three wooden board in size 40 cm × 40 cm.
2. Fix two in such a way that they intersect or thogonally in the middle as shown in the figure i.e. along YY´ & ZZ´.
3. Cut the third sheet into two equal rectangles.
4. Insert another sheet along XX^1 so that three planes can intersect at right angles at a point (say O) and divide eight parts.
5. Fix all sheet by nails/wires.



Demonstration :

1. XX^1, YY^1, ZZ^1 shows x-axis, y-axis, z-axis.
2. All the axis intersect at O i.e. origin.
3. XOZ, XOZ^1, X^1OZ, X^1OZ^1, OYZ, OY^1Z, OYZ^1, OY^1Z^1, XYO, XY^1O, X^1YO, X^1Y^1O are shows different planes.
4. The co-ordinates of any point in 3D is triplet i.e. (x, y, z).

Activity - 21

Objective : To Understand relation and functions.

Learning Outcome : From the activity students will be able to understand Relations and functions.

Materials Required : Hard board, 1V and 2V bulbs, testing screws, tester, electrical wire and switches, battery.

Method of Construction :

1. Take a Hard board.
2. Drill eight holes along first column on one side of the board A, B, C, D, E, F, G and H as shown in figure 1.
3. Drill nine holes on other side of the board P, Q, R, S, T, U, V, W and X as shown in the figure 1.

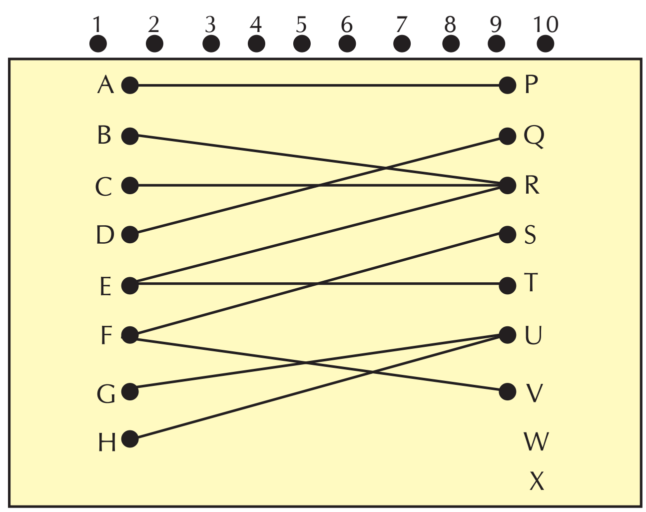


Figure : 1

4. Fix 1V bulbs in A, B, C, D, E, F, G and H.
5. Fix 2V bulbs in P, Q, R, S, T, U, V.
6. Fix 10 testing screws as shown in fig : 1.

Demonstration :

1. Bulbs along the first column represent domain.
2. Bulbs along the second coloumn represent co-domain.
3. Bulbs along the second column except W,X is called the image.
4. Ordered pairs (A, P), (B, R), (C, R), (D, Q), (E, R), (F, S), (E, T) etc. represents elements of a relation.

Observation :

1. A function maps {A, B, C, D, E, F, G, H} to {P, Q, R, S, T, U, V}
2. Function is one Many function and Many one also.
3. Range of the function is { P, Q, R, S, T, U, V}
4. Any bulb in the second column (except W, X) is called the image which defines the inverse element.

Activity - 22

Objective : To explain the concept of discontinuity of a function.

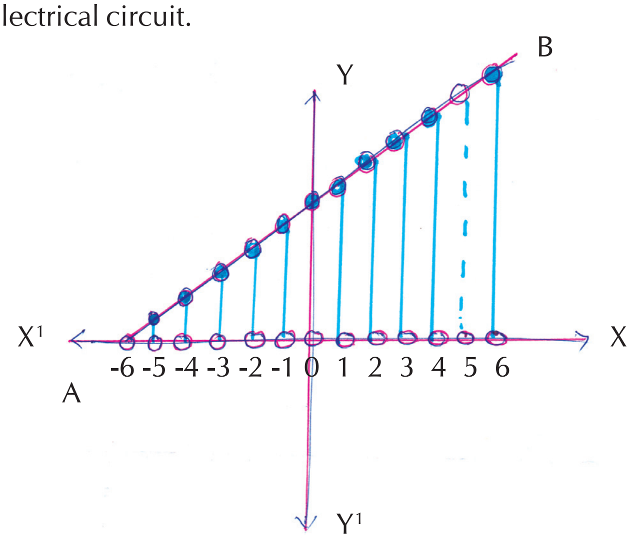
Learning Outcome :

1. For this model student will be able to understand the defineness of the function.
2. Also the student will be able to learn the discontinuity/continuity of a function.

Materials Required : Hard board, 1Volts bulbs, one 2 Volt bulbs, testing screws, electrical circuit.

Preparation :

1. Take a Hard board of size 40cm × 40 cm.
2. Drill holes along horizontal line at equal lengths of interval and mark as –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6 and fix bulbs in those holes.
3. Drill holes along the line which is inclined at an angle 45° with the horizontal line. Fix bulbs in those holes.
4. Complete the electrical circuit.



Demonstration :

1. Bulbs along the horizontal line represent the points on the X axis.
2. Bulbs along the line AB represents the corresponding points on the graph of $y = \frac{x^2-25}{x-5}$
3. At $x = 5$ the bulb does not glow
4. In the neighbourhood of $x = 5$ the bulbs glows.
5. The existence of left hand and right hand limits can be seen by glowing other bulbs.

Observation :

1. At $x = 5$ the function is not defined
2. The function $y = \frac{x^2-25}{x-5}$ is continuous except $x = 5$
3. At $x = 5$ the function is discontinuous.

Activity - 23

Objective : To explain the graph of the function.

$f(x) = e^x$ and $f(x) = \log_e x$

Learning Outcome :

By using this model the students will be able to learn the properties of the exponential and logarithmic function.

Materials Required : Wooden strips, wire, Colour.

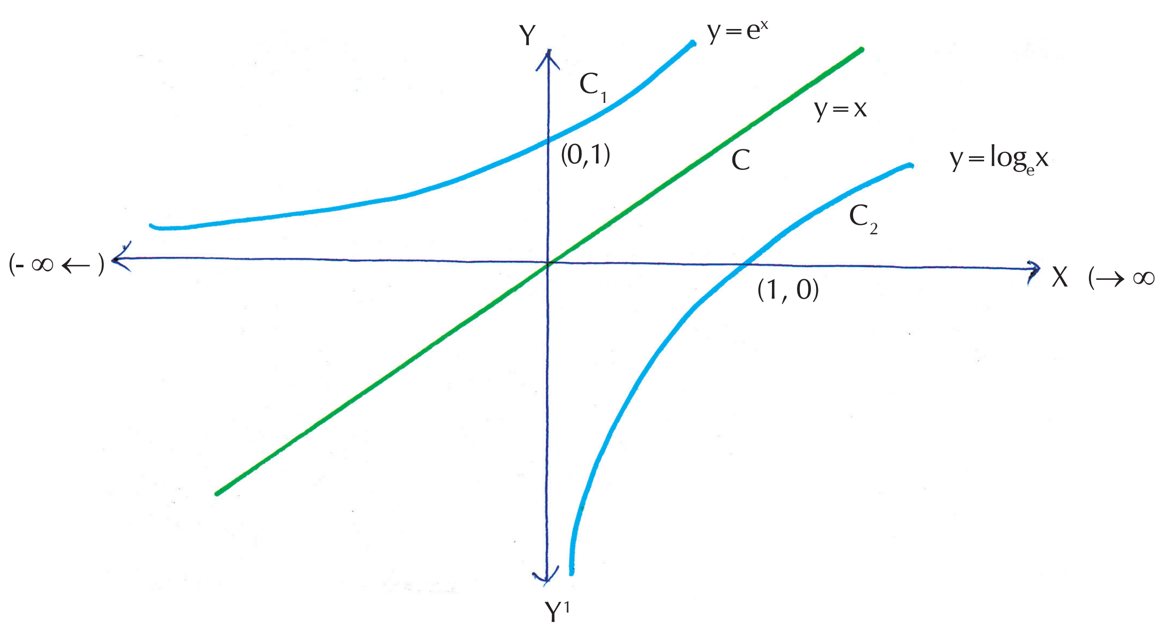
Preparation :

1. Fix two perpendicular strips intersecting at a point to represent x-axis and y-axis.
2. Draw the graph of e^x by using the following values of x and e^x. :

x	–3	–2	–1	0	1	2	3
e^x	0.05	.135	.3	1	2.7	7.8	20.1

3. Draw the graph of $\log_e x$ by using the following values of x and $\log_e x$

x	.04	.1	.2	.5	1	e
$\log_e x$	–3.2	–2.3	–1.6	–.69	0	1



4. Take wires in the shape of the graphs of e^x and $\log_e x$ and fix them on the wooden boards.
5. Colour the wires to represent different curve .

Demonstration :

1. C_1 represents the curve of $y = e^x$
2. C_2 represents the curve of $y = \log_e x$

Observation :

1. The curve of $y = e^x$ crosses y axis at ________.
2. The curve of $y = \log_e x$ crosses x-axis at ________.
3. The line (green colour) represents the curve of ________.
4. The graphs of e^x and $\log_e x$ are symmetric about the line ________.
5. $e^x =$ ________ when $x \to -\infty$, $e^x =$ ________ when $x \to \infty$.
5. $\log_e x$ is defined when x is lies between ________.

Activity - 24

Objective : To draw the graphs of sin x and $\sin^{-1}x$.

Learning Outcome :

From this model / activity students will be able to explain the graph of sin x and $\sin^{-1}x$.

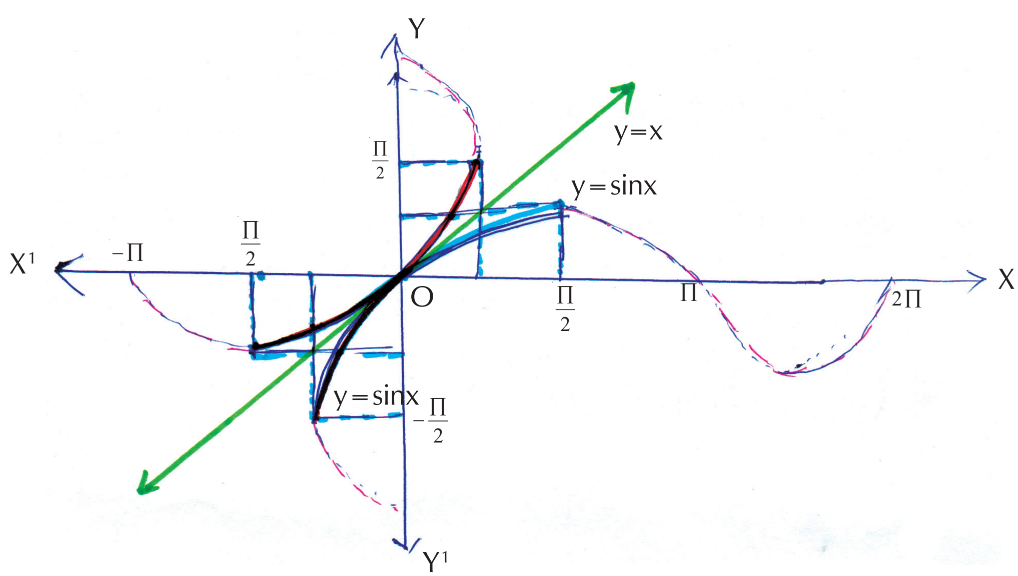
Materials Required : Wooden strips, wire, soldering wire

Preparation :

1. Fix two wooden strips perpendicular to each other and intersecting at O to represent X-axis and Y-axis.
2. Draw the graph of sin x with the values given below :

x	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π
sin x	0	–1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	1	0
y		–1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	1	
sin x		$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	

3. Take up three wires in the shape of sinx, $\sin^{-1}x$ and y = x which fix-up them on the wooden strip.
4. Paint the wires with different colours.



Demonstration :

1. The graph of sin x crosses x-axis at the points .... (–2π, 0), (–π, 0), (0,0), (π, 0),(2π, 0,................
2. The graph of $\sin^{-1}x$ crosses y-axis at the points ..... (0, –2π), (0, –π), (0, 0), (0, π), (0, 2π), ...

Activity - 25

Objective : To verify Rolle's theorem. (Say, $f(x) = \sin x + 2, 0 \le x \le 2\pi$)

Learning Outcome :

From this theorem students will be able to find the roots of an equation (Polynominal, Trigomometric, exponential functions etc.

Materials Required : Cardboard, wire, white paper, adhesive, pin, scale, sketch pen etc.

Method of Construction :

1. Take a cardboard of a convenient size & paste a white paper on it.
2. Take two wires of convenient size & fix them with the help of pin on the white paper pasted on cardboard to represent x-axis & y-axis (see Fig. 1).
3. Take 20cm length of wire & bend it in the shape of a curve & fix it with the help of pin on the card board as shown in figure-1.
4. Take three straight wires of the same length & fix them in such way that they are perpendicular to x-axis at the points O, O_1 & O_2 meeting the curve at the points C, C_1 & C_2 respectively.

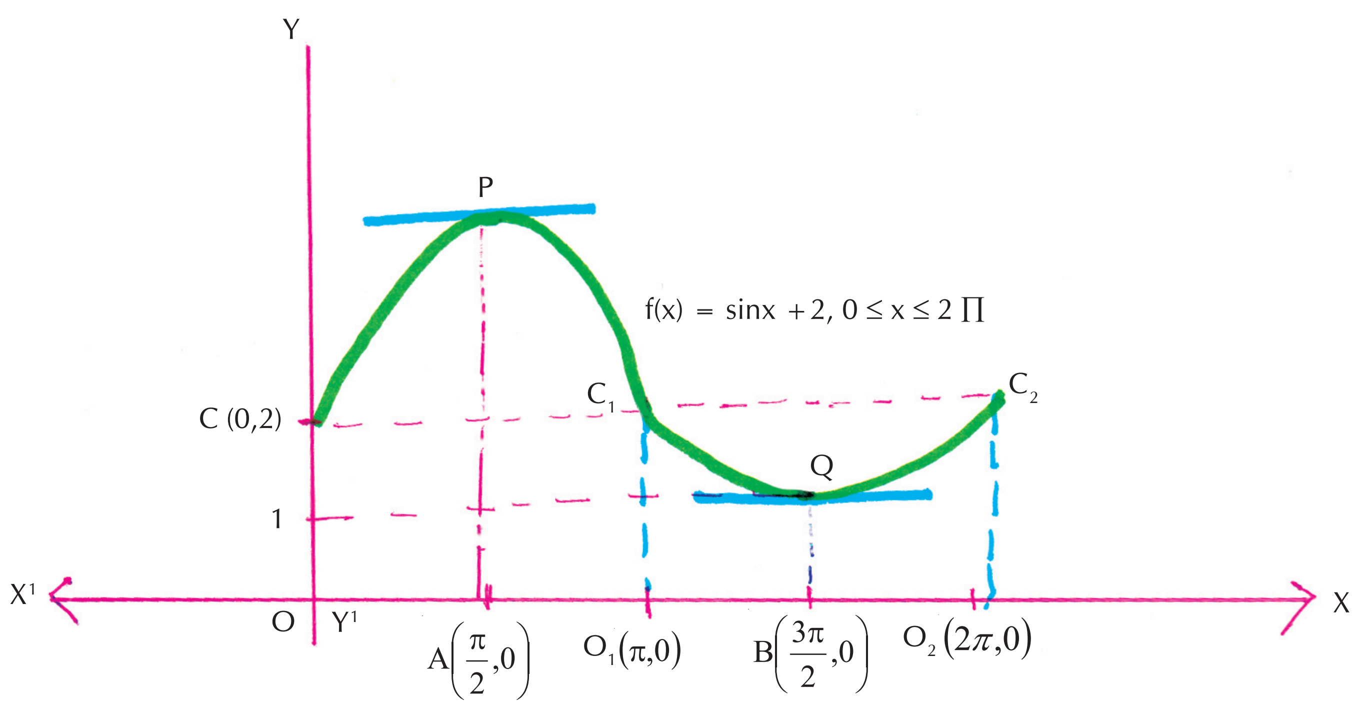


fig-1

Demonstration :

1. In the fig-1, let the curve represent the function $y = f(x) = \sin x + 2, 0 \le x \le 2\pi$. Let $OO_1 = \pi$ & $OO_2 = 2\pi$.
2. The Co-ordinates of the points O, O_1 & O_2 are (0, 0), (π, 0) & (2π, 0) respectively.

3. Clearly in figure, $f = \sin x + 2, 0 \le x \le 2\pi$ is continuous in $[0, 2\pi]$
4. For smoothness of the curve in $[0, 2\pi]$ a tangent can be drawn which gives that the function f is differentiable on $(0, 2\pi)$.
5. Here, $OC = OC_1 = OC_2$

   So, $f(0) = f(\pi) = f(2\pi)$
6. From steps (3), (4) & (5), it is clear that Rolle's theorem are verified.

Also, from fig.-1, we see that tangents at P & Q are parallel to x-axis, therefore, $f'(x) = 0$ at P & Q.

Thus, there exists (∃) at least one value $\frac{\pi}{2}$ of x in $(0, \pi)$

such that, $f'(\frac{\pi}{2}) = 0$ & also $\frac{3\pi}{2}$ in $(\pi, 2\pi)$

such that, $f'(\frac{3\pi}{2}) = 0$.

Hence, Rolle's theorem is verified.

Observation :

From Fig. -1

In $0 \le x \le \pi$,

$f(0) =$ ________ , $f(\pi) =$ ________ .

Is $f(0) = f(\pi)$? (Yes/No)

Slope of tangent at P = ________ , So, f(x) (at P) = ________ .

In $\pi \le x \le 2\pi$,

$f(\pi) =$ ________, $f(2\pi) =$ ________

Is $f(\pi) = f(2\pi)$? (Yes/No)

Slope of tangent at Q = ________ ,

So, f(x) (at Q) = ________ .

In $0 \le x \le 2\pi$,

$f(0) =$ ________ , $f(2\pi) =$ ________ .

Is $f(0) = f(2\pi)$? (Yes/No)

Slope of tangent at P & Q = ________ ,

So, f(x) (at P & Q) = ________ .

Activity - 26

Objective : To understand the concept of Local Maxima & Local Minima.

Learning Outcomes :

1. From this activity students will be able to understand the concepts of Local Maxima & Local Minima.
2. This concepts may be used in linear programming problem (LPP) for maximum capacity at minimum cost.

Materials Required : Cardboard, wire, white paper, adhesive, pin, scale, sketch, pen etc.

Method of Construction :

1. Take a cardboard of a convenient size & paste a white paper on it.
2. Take two wires of convenient size & fix them with the help of pin on white paper pasted on cardboard to represent x-axis & y-axis (see Fig.-1)
3. Take 50 cm length of wire & bend it in the shape of a curve & fix it with the help of pin on the cardboard as shown in fig.-1.
4. Fix the wire at the points A, B, C, D & E as shown in figure.

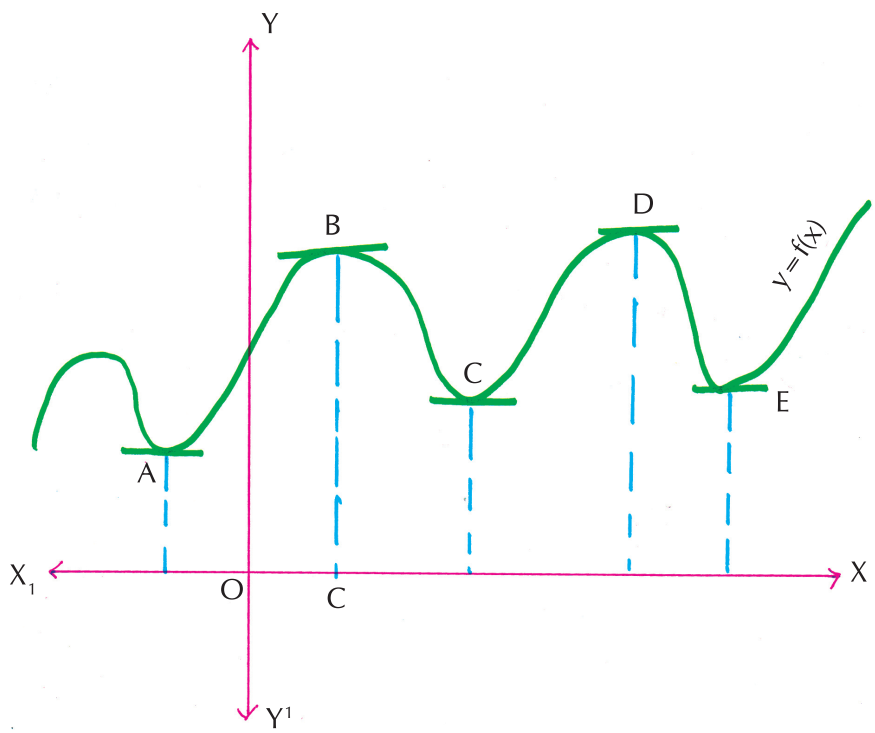


fig-1

Demonstration :

1. In fig-1, at the point B represent tangent to the curve y = f(x) and it is parallel to x-axis. The slope of tangent at the point are zero i.e. $f'(c) = 0$
2. At the points B & D, sign of 1st derivative changes from +ve to –ve. So, they are the points of local maxima.

3. At the points C & E, sign of 1st derivatives changes from - ve to +ve. So, they are the points of local minima.
4. In fig.1, Local Maxima & Minima occur only at points where the tangent is horizontal (i.e., $f'(c) = 0$) or when there is no tangent line (i.e., $f'(x)$ does not exists).

Observation :

1. Sign of the slope of the tangent (1st derivative) at a point on the curve to the immediate left of B is ________________.
2. Sign of the slope of the tangent (1st derivative) at a point on the curve to the immediate right of B is ________________.
3. Sign of derivative at a point immediate left of C is ________________ & immediate right of C is ________________ .
4. Sign of derivative at a point immediate left of D is ________________ & immediate right of D is ________________ .
5. Sign of derivative at a point immediate left of E is ________________ & immediate right of E is ________________.
6. B & D are points of local ________________ .
7. C & E are points of local ________________ .

Activity - 27

Objective : To understand the concept of tangent to a curve using calculas.

Learning Outcome :

From this activity students will be able to understand the concept of tangent to a curve.

Materials Required : Hardboard, White Paper, Graph Paper, Pencil, Scale, Adhesive, Calculator

Method of Construction :

1. Paste a graph paper on a white paper & fix the paper on the hardboard.
2. Draw the graph of a curve on the graph paper (See Fig. - 1)

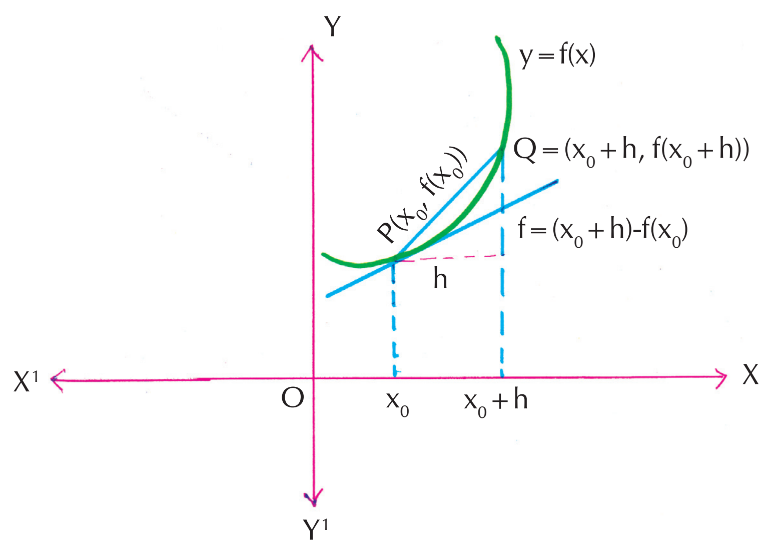


Figure : 1

3. Using calculator & draw the curves $y = x^2$ & $y = x^3$ on the graph paper (See Fig - 2 & 3 respectively)

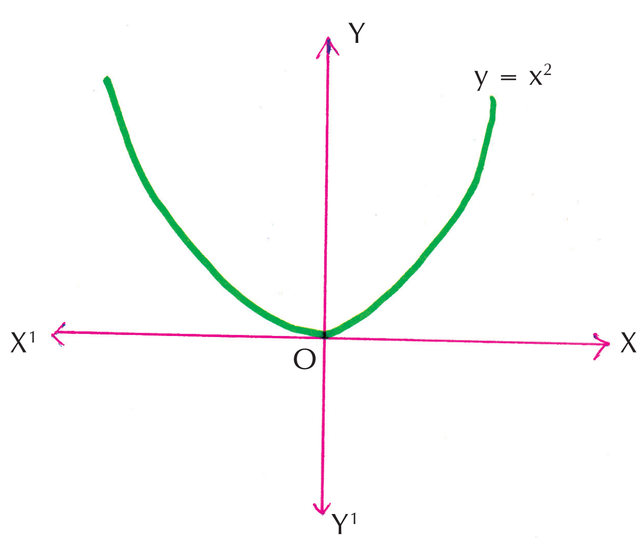


Figure : 2

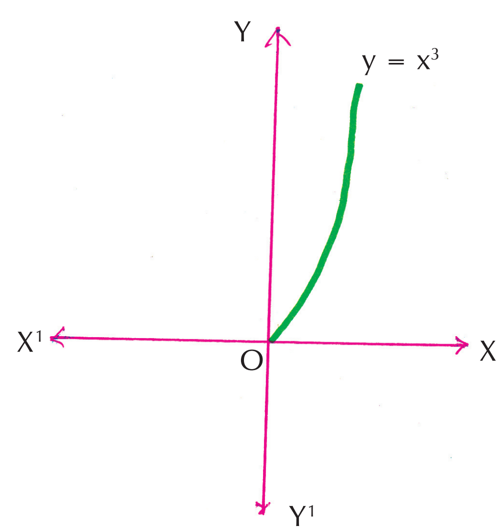


Figure : 3

4. Using calculator & draw the curves $y = x^{2/3}$ & $y = x^{1/3}$ on the graph paper (See Fig - 4 & Fig - 5 respectively)

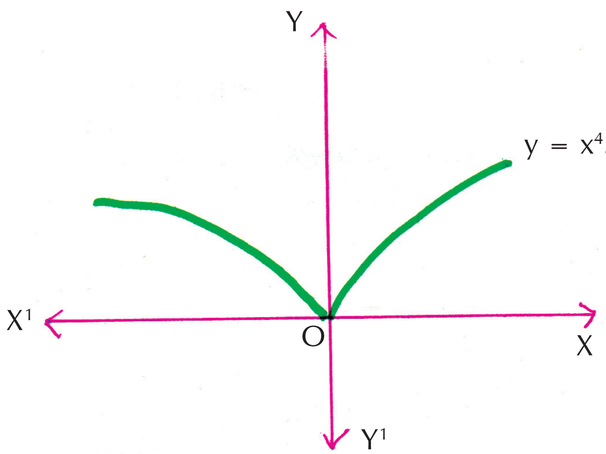


Figure : 4

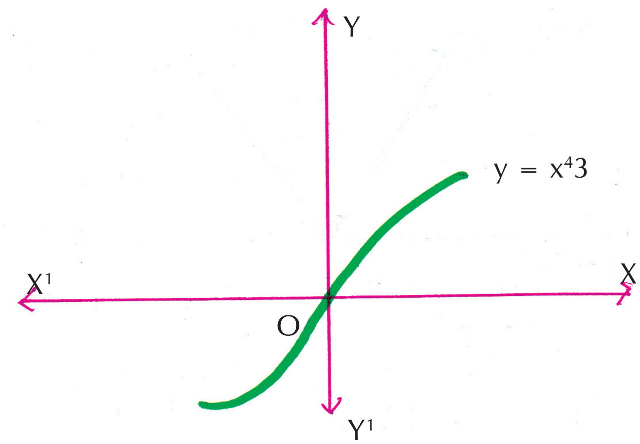


Figure : 5

5. Using calculator & draw the curves $y = |x|$ on the graph paper (See Fig - 6)

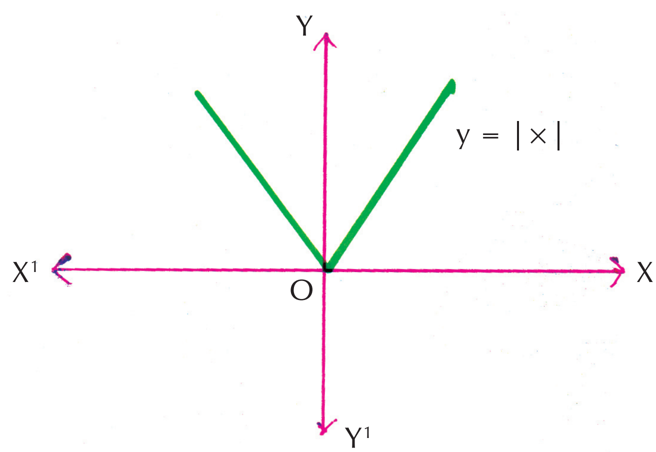


Figure : 6

Demonstration :

1. In Fig - 1, take any two neighbouring points $P(x_o, f(x_o))$ & $Q(x_o + h, f(x_o + h))$.

2. Slope of the curve $y = f(x)$ at the point $P(x_o, f(x_o))$ is the number $m = f'(x_o) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$ (Provided the limit exists).

3. The tangent line to the curve at P is the line through P with this slope, so, the equation of tangent at P is given by $y-y_o = m(x-x_o)$

4. For this value of x, find the value of y - coordinates for all graphs $y = x^2$, $y = x^3$, $y = x^{2/3}$, $y = x^{1/3}$ & $y = |x|$ by actual measurement, using scale & mark them.

5. Find the slope m.

6. Find the tangent lines of all curves if it exists.

Observation :

Sl. No.	Curves	Slope	Tangent Equation
1.	$y = x^2$	At the Pt. (0,0) $m = \lim_{n\to 0} \frac{f(h) - f(0)}{n}$ $= \lim_{h\to 0} h^2/h = 0$	$y - 0 = m(x-0)$ or, $y = 0$
2.	$y = x^3$	At the Pt. (0,0) $m = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$ $= \lim_{h\to 0} h^3/h = \lim_{h\to 0} n^2 = 0$	$y - 0 = 0\ (x - 0)$ or, $y = 0$
3.	$y = x^{2/3}$	At the Pt. (0,0) $m = \lim_{h\to 0+} \frac{f(h) - f(0)}{h}$ $= \lim_{h\to 0} 1/h^{1/3} = \infty$	Hence, there is no vertical tangent to the curve $y = x^{\frac{2}{3}}$ at (0,0)
4.	$y = x^{1/3}$	At the Pt. (0,0) $m = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \infty$ Similarly, $\lim_{h\to 0} -\frac{f(h) - f(0)}{h} = -\infty$	So , the vertical tangent to the curve $y = x^{\frac{1}{3}}$ at (0,0) is $x = 0$
5.	$y = \|x\|$	At the Pt. (0,0) $f'(0+0) = 1$ & $f'(0-0) = -1$ Moreover, $\lim_{h\to 0}$ if $\left\| \frac{f(h) - f(0)}{h} \right\| = 1 \neq \infty$	Thus, $y = \|x\|$ has an undefined slope. No tangent line to its tangent thread.

Activity - 28

Objective : To understand the concept of normal to a curve using calculas.

Learning Outcomes :

From this activity students will be able to understand the concept of normal to a curve.

Materials Required : Hardboard, white paper, graph paper, pencil, scale, adhesive, calculator.

Method of Construction :

1. Paste a graph paper on a white paper & fix the paper on the hardboard.
2. Draw the graph of a curve on the graph paper (See Fig. - 1)

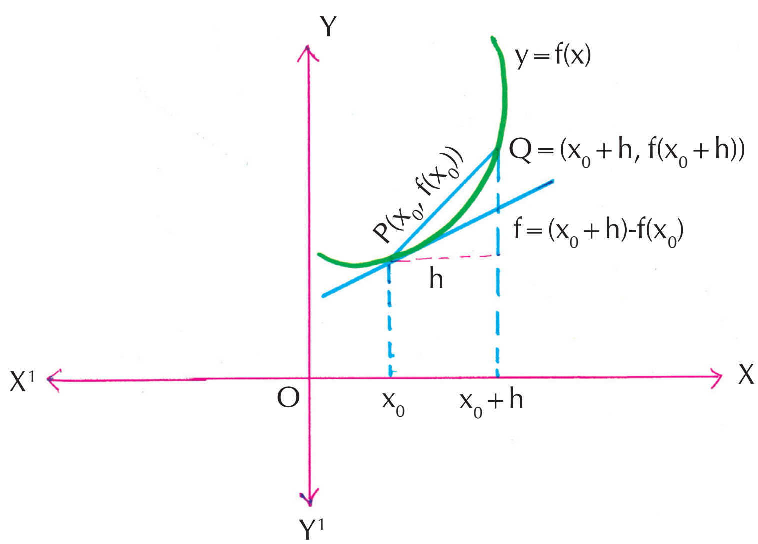


Figure : 1

3. Draw the equations of conics on graph paper viz —
   Circle : $x^2 + y^2 = a^2$, $x^2 + y^2 + 2gx + 2fy + c = 0$
   Parabola : $y^2 = 4ax$

   Ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

   Hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Demonstration :

1. In Fig. - 1, take any two neighbouring points P $(x_o, f(x_o))$ & Q $(x_o+h, f(x_o+h)$ namely.

2. Slope of the curve $y=f(x)$ at the point $P(x_o, f(x_o))$ is the number

$$m = f'(x_o) = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \text{ (provided the limit exists)}$$

3. The tangent line to the curve at P is the line through P with this slope, so the equation of tangent at P is given by $y-y_0 = m(x-x_0)$.

4. We know that the line through (x_0, y_0) perpendicular to the tangent line is called the normal line.

   Since the slope of the tangent is $f'(x_0)$, the slope of normal is $-\frac{1}{f^1(x_0)}$ provided $f'(x_0) \neq 0$.

   So, the equation of normal is $y - y_0 = -\frac{1}{m}(x-x_0)$

5. Find the normal lines of all curves if it exists.

Observation :

Sl. No.	Equation of Conic	Equation of tangent	Equation of normal
1.	Circle : $x^2 + y^2 = a^2$	$xx_1 + yy^1 = a^2$	$xy_1 - yx_1 = 0$
2.	Circle : $x^2 + y^2 + 2gx + 2fy + c = 0$	$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) = 0$	$(x-x_1)(y_1+f) = (y-y_1)(x_1 + g)$
3.	Parabola : $y^2 = 4ax$	$yy_1 = 2a(x + x_1)$	$y_1(x-x_1) + 2a(y-y_1)=0$
4.	Ellipse : $x^2/a^2 + y^2/b^2 = 1$	$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$	$\frac{x-x_1}{\frac{x_1}{a_2}} = -\frac{y-y_1}{\frac{y_1}{b_2}}$
5.	Hyperbola : $x^2/a^2 - y^2/b^2 = 1$	$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$	$\frac{x-x_1}{\frac{x_1}{a_2}} = -\frac{y-y_1}{\frac{y_1}{b_2}}$

Activity - 29

Objective : To understand the concept of definite integral $\int_a^b f(x)dx$.

Learning Outcomes :

1. Learners will be able to understand the concept of definite integral $\int_a^b f(x)dx$.
2. Student will be able to demonstrate the concept of area bounded by a curve.
3. From this activity student will be able to understand the meaning of $\int_a^b f(x)dx$.
4. Learners is able to demonstrate the concept of area bounded by a curve.

Materials Required : Geoboard, few different colours rubber bands.

Method of Construction :

1. Take a geoboard of convenient size with given co-ordinate axes XOX′ and YOY′
2. Use green rubber band to make the graph (CD) of the function y = f(x) defind in the interval $a \leq x \leq b$ where a & b are finite and A, B represents the points x = a and x = b respectively ; then $\overline{OA} = a$; $\overline{OB} = b$ and $AB = \overline{OB} - \overline{OA} = b - a$. Again, if $\overline{AC}$ and $\overline{BD}$ are the ordinates at A & B respectively, then AC = f(a) and $\overline{BD}$ = f(b). (See Fig - 1)

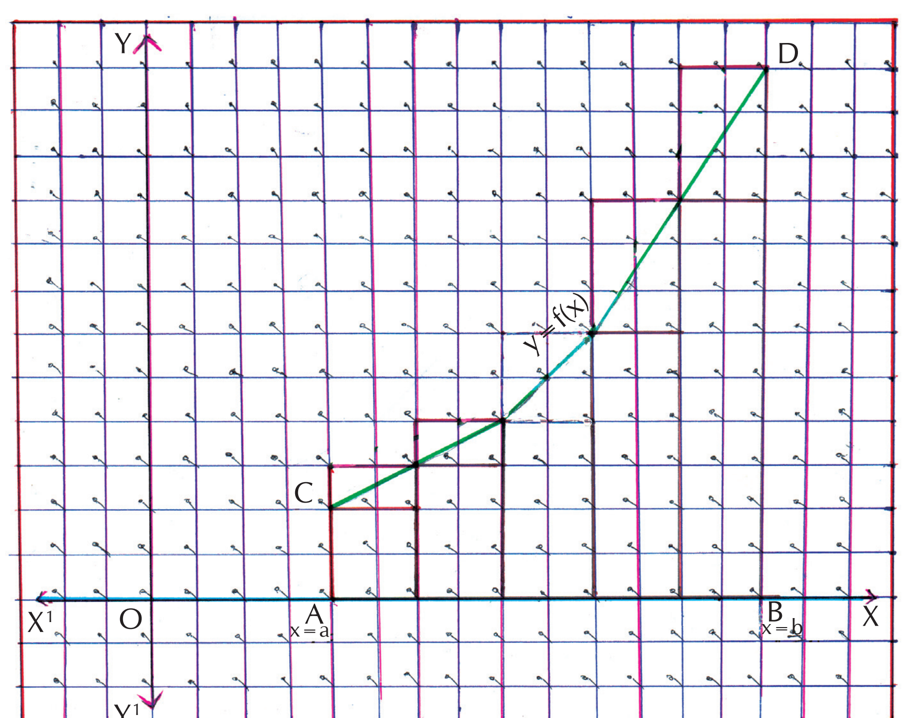


Figure : 1

3. Now divide the line segment AB in to n equal parts each of length h by the points a, (a+x)...., {a + (n–1)h}, a +nh; then a + nh = b or nh = b – a. Erect ordinates at each of the points a + h, a + 2h,..., a + (n–1)h and complete the inner and outer rectangles using rubber band. [See Fig - 1]

Demonstration :

1. Let S be the area bounded by the curve y = f(x), the axis, the ordinate x = a and the ordinate x = b and S_1, S_2, be the sums of the areas of the inner and outer rectangles respectively. Then, from Fig 1 it is evident that, $S_1 < S < S_2$

2. $S_1 = hf(a) + hf(a + h) + ... + hf\{a + (n-1)h\}$

$$= h\sum_{r=0}^{n-1} f(a + rh)$$

and $S_2 = hf(a+h) + h f(a + 2h) + hf(a + 2h) + ... + hf(a+n)$

$$= h f(a) + h f(a+h) + h f(a+2h) + ... + hf\{a+(n-1)h\} + h f(a+nh) - hf(a)$$

$$= h\sum_{r=0}^{n-1} f(a + rh) + hf(b) - hf(a) \; [\because a + nh = b]$$

3. Now, a and b are finite and nh = b – a or, $h = \frac{b-a}{n}$

Hence, $h \to 0$, when n gradually increases and ultimately tends to infinity. Again f(a) and f(b) both are finite.

Therefore, hf(a) $\to$ 0 and hf(b) $\to$ 0, when $h \to 0$

Hence, $\lim_{h\to 0} S_2 = \lim_{h\to 0} [h\sum_{r=0}^{n-1} f(a+rh) + hf(b) - hf(a)]$

$$= \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a + rh) + \lim_{h\to 0} hf(b) - \lim_{h\to 0} hf(a)$$

$$= \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a + rh) = \int_a^b f(x)dx$$

and $\lim_{h\to 0} S_1 = \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a + rh) = \int_a^b f(x)dx$

i.e when $h \to 0$, then then $S_1 \to \int_a^b f(x)dx$ and $S_2 \to \int_a^b f(x)dx$

But, $S_1 < S < S_2$

Therefore, when $h \to 0$ we have

$$S = \int_a^b f(x)\,dx$$

Hence, geometrically the area bounded by the curve $y = f(x)$, the x-axis, the ordinate $x = a$ and the ordinate $x = b$ represents the definite integral $\int_a^b f(x)dx$ and conversely.

Observation :

1. When $h \to 0$, then S_1 tends to ______________ .
2. When $h \to 0$, then S_2 tends to ______________.
3. When $h \to 0$, we have, $S =$ ______________.

Activity - 30

Objective : To evaluate the definite integral $\int_0^4 \frac{5x}{4}\,dx$ using the concept of area and verify it by actual integration.

Learning Outcome :

From this activity students will be able to demonstrate the concept of area bounded by straight line and x-axis, y-axis.

Materials Required : Geoboard, few rubber bands/threads

Method of Construction :

1. Take a geoboard of convenient size with given co-ordinate axes XOX′ and YOY′.
2. Use rubber band/thread to make the straight line $y = \frac{5x}{4}$ as shown in Fig.1.
3. Again use the rubber band/thread to make the ordinate $x = 4$. (the line PQ as shown in Fig. 1) and use another rubber band to denote the line OP. (Fig - 1)

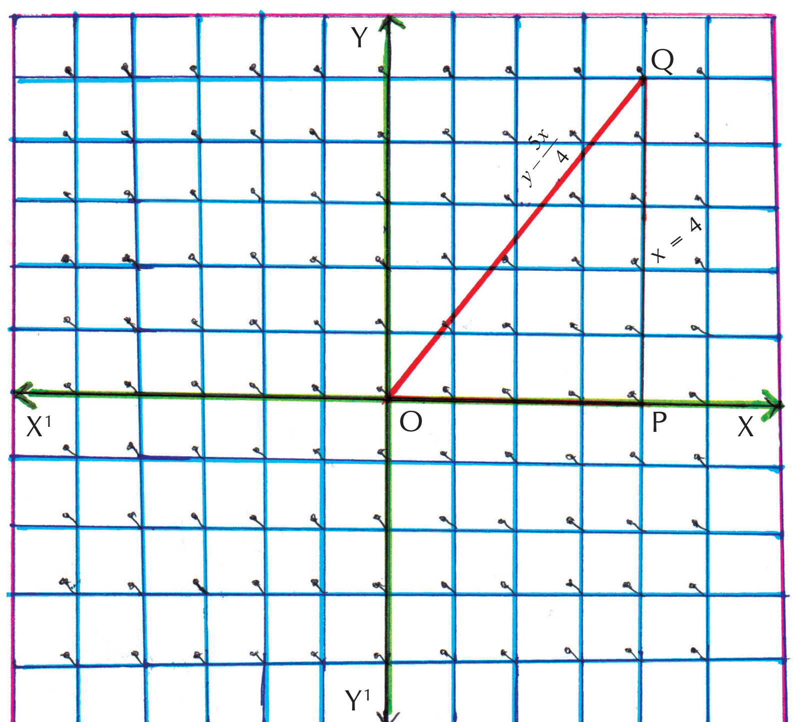


Figure : 1

Demonstration :

1. The area bounded by the straight line $y = \frac{5x}{4}$, x - axis and the ordinate $x = 4$ form a right-angled traingle OPQ.

2. To find the area of the triangle OPQ, count the number of unit squares involved. The area of each complete square is counted as 1sq.cm and that of each incomplete square is taken to be $\frac{1}{2}$ sq.cm.

3. Calculate the area of the triangle by counting unit squares —

   Number of complete unit squares = 6

   Number of incomplete unit squares = 8

$\therefore$ Area of the triangle OPQ $= \{(6\times 1) + (8\times \frac{1}{2})$ sq.cm

$= (6 + 4)$ sq.cm

$= 10$ sq.cm

4. Calculate the area of the triangle using formula —

   Area of the triangle OPQ $= \frac{1}{2}\times OP\times PQ$

   $= \frac{1}{2}\times 4\times 5$

   $= 10$ sq.cm

5. Definite Integral $= \int_0^4 \frac{5x}{4}dx$

   $= \frac{5}{4}\left[\frac{x^2}{2}\right]_0^4$

   $= \frac{5}{4}[8-0] = 10$ sq.cm

Thus the area of the triangle is same as the area obtained by actual integration.

Observation :

1. Area of the triangle obtained by counting unit squares = ____________cm^2
2. Area of the triangle obtained by using formula = ___________ cm^2
3. Area of the bounded region $= \int_0^4 \frac{5x}{4}dx =$ __________.
4. The three areas are ___________.

Activity - 31

Objective : To find the area of the first quadrant bounded by the circle $x^2 + y^2 = 9$, the x-axis and the ordinates $x = 1$ and $x = 2$ as the limit of a sum and verify it by actual integration.

Learning Outcomes :

From this activity students will be able to demonstrate the concept of area bounded by a curve and to find the approximate value of π .

Materials Required : Cardboard, white paper, scale, pencil, glue.

Method of Construction :

1. Take a cardboard of convenient size and paste a white paper on it.
2. Draw two perpendicular lines to represent co-ordinate axes XOX′ and YOY′.
3. Draw a quadrant of a circle with O as centre and radius 3 unit (6 cm) as shown in Fig - 1. The area of the 1st quadrant bounded by the circle $x^2 + y^2 = 9$, the x-axis and the ordinates $x = 1$ and $x = 2$.

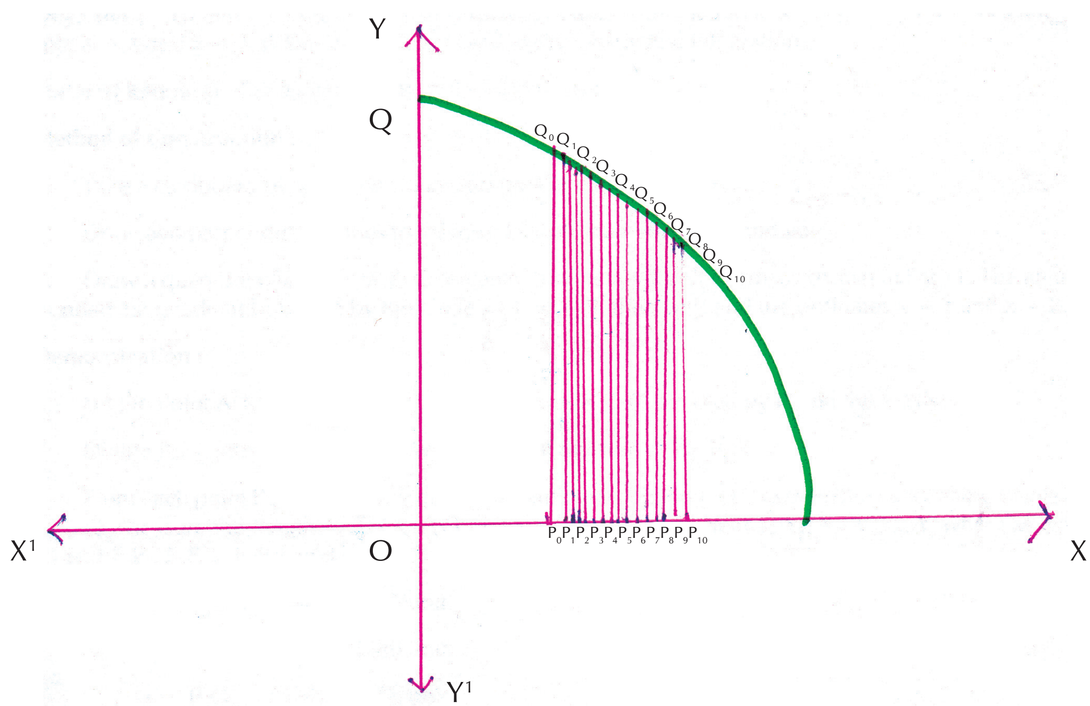


Figure : 1

Demonstration :

1. Let the Point A(1,0) denoted by P_0 and the points B (2,0) denoted by P_{10} on the x-axis.
2. Divide P_0P_{10} into 10 equal parts with points of division as $P_1, P_2, P_3, \ldots, P_9$
3. From each point P_i, $i = 1,2,3\ldots,9$ draw perpendiculars on the x-axis to meet the curve at the points Q_1,

Q_2, Q_3..., Q_9.Measure the lengths of P_0Q_0, P_1Q_1, P_2Q_2,...P_9Q_9 and call them as y_0, y_1,....y_{10} where as width of each part P_0P_1, P_1P_2....is 0.1 units.

4. $y_0 = P_0Q_0 = 2.85$ unit

$y_1 = P_1Q_1 = 2.80$ unit

$y_2 = P_2Q_2 = 2.75$ unit

$y_3 = P_3Q_3 = 2.70$ unit

$y_4 = P_4Q_4 = 2.65$ unit

$y_5 = P_5Q_5 = 2.60$ unit

$y_6 = P_6Q_6 = 2.55$ unit

$y_7 = P_7Q_7 = 2.50$ unit

$y_8 = P_8Q_8 = 2.40$ unit

$y_9 = P_9Q_9 = 2.30$ unit

$y_{10} = P_{10}Q_{10} = 2.25$ unit

5. Area of the first quadrant bounded by the circle $x^2 + y^2 = 9$, the x-axis and the ordinates $x = 1$ and $x = 2$

= Sum of the area of trapesiums.

$= \frac{1}{2} \times 0.1\ [(2.85 + 2.80)] + (2.80 + 2.75) + (2.75 + 2.70)\ (2.70 + 2.65) + (2.65 + 2.60) +$

$(2.60 + 2.55) + (2.55 + 2.50) + (2.50 + 2.40) + (2.40 + 2.30) + (2.30 + 2.25)]$

$= \frac{1}{2} \times 0.1 \times [5.65 + 5.55 + 5.45 + 5.35 + 5.25 + 5.15 + 5.05 + 4.90 + 4.70 + 4.55]$

$= \frac{1}{2} \times 0.1 \times 51.6$

$= 0.1 \times 25.8$

$= 2.58$

6. Definite integral $= \int_1^2 \sqrt{9 - x^2}\,dx$

$= \left[\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_1^2$

$$= \left(\frac{2\sqrt{9-4}}{2}+\frac{9}{2}\sin^{-1}\frac{2}{3}\right)-\left(\frac{1\sqrt{9-1}}{2}+\frac{9}{2}\sin^{-1}\frac{1}{3}\right)$$

$$= \sqrt{5} + 4.5\times 0.72 - \sqrt{2} - 4.5\times 0.34$$

$$= 2.236 + 3.24 - 1.414 - 1.53$$

$$= 5.476 - 2.944$$

$$= 2.532$$

Thus, the area of the first quadrant bounded by the circle $x^2 + y^2 = 9$, the x-axis and the ordinates $x = 1$ and $x = 2$ as a limit of a sum is nearly same as area obtained by actual integration.

Observation :

1. Area of the bounded region $= \int_1^2 \sqrt{9-x^2}\,dx = =$ ___________ sq. unit.

2. Area of the bounded region as a limit of a sum = ___________ sq. unit.

3. The two area are nearly ___________.

Activity - 32

Objective : To evaluate the definite integral verify $\int_0^{\pi} \sin x \, dx$ as the area of the bounded region and verify it by actual integration.

Learning Outcomes :

1. From this activity students will be able to demonstrate the concept of area bounded by a curve.
2. From this activity students will be able to evaluate the integral $\int_0^{\pi/2} \sin x dx$, $\int_0^{\pi/2} \cos x dx$ etc.

Materials Required : Cardboard, scale, pencil, sketch pen, large graph, grid paper.

Method of Construction :

1. Take a cardboard of convenient size and paste a grid paper on it.
2. Draw two perpendicular lines to represent Co-ordinate axes XOX′ and YOY′.
3. Draw the graph of the curve y = sinx from the origin O as shown in Fig. 1.
4. Shade the region as shown in Fig.1.

The Shaded region bounded by the function y = sinx in the interval [0, π] and x-axis.

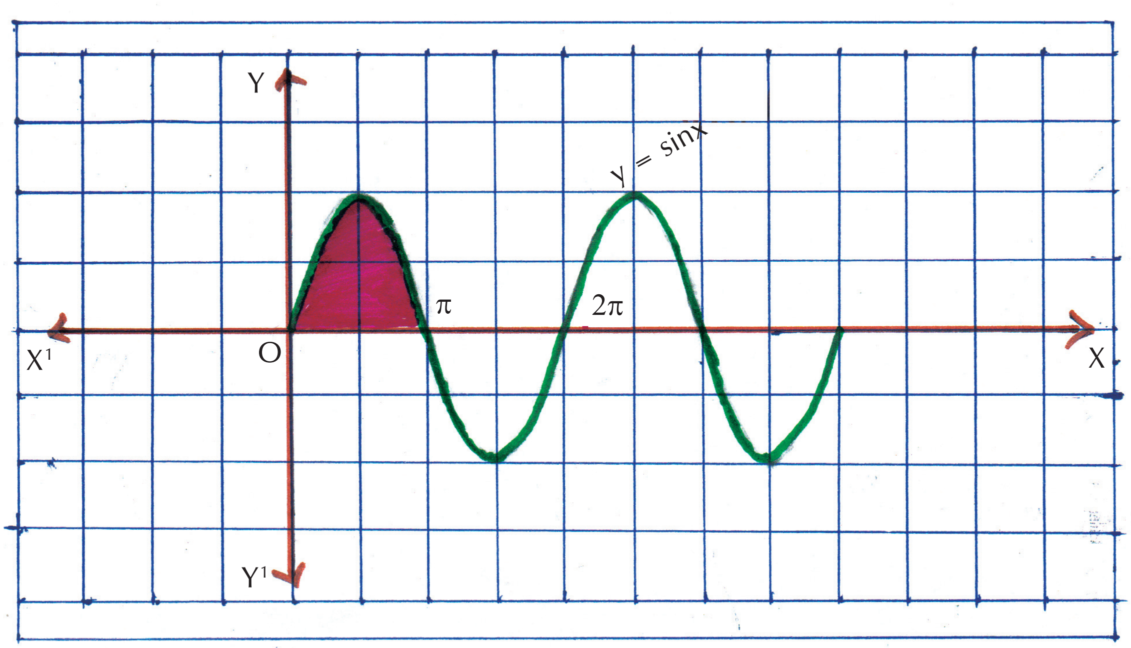


Figure : 1

Demonstration :

1. To find the area of the shaded region, count the number of unit squares involved. The area of each complete square is counted as 1 sq.unit and that of each incomplete square is taken to be $\frac{1}{2}$ sq. unit.

2. Calculate the area of the shaded region by counting unit squares —

   Number of complete unit square = 0

   Number of incomplete unit square = 4

   ∴ Area of the shaded region $= \{(0 \times 1) + (4 \times \frac{1}{2})\}$sq.unit.

   = 2 sq.unit.

3. Definite integral $= \int_0^{\pi} \sin x dx$

   $= [-\cos x]_0^{\pi}$

   $= -[\cos \pi - \cos 0]_0^{\pi}$

   $= -[-1-1]$

   = 2 sq. unit.

Thus the area of the shaded region obtained by counting unit square is same as the area obtained by actual integration.

Observation :

1. Area of the shaded region by counting unit square = ___________ sq.unit.

2. Area of the shaded region obtained by actual integration $= \int_0^{\pi} \sin x dx =$ ___________ sq. unit.

3. The two areas are ___________.

Activity - 33

Objective : To establish $|x| = |-x|$

Learning Outcomes :

1. From this activity students will be able to compare graph of $|x|$ & $|-x|$.
2. Students will be able to understand absolute valued function.

Materials Required : Geoboard, pencil, eraser, rubber band etc.

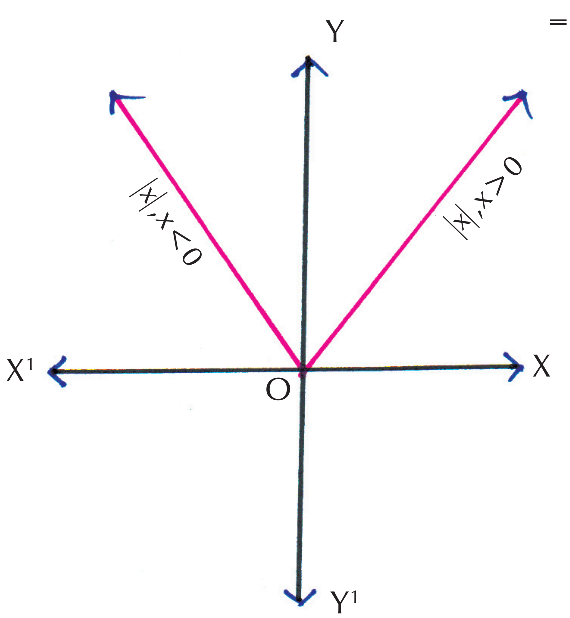
Method of Construction :

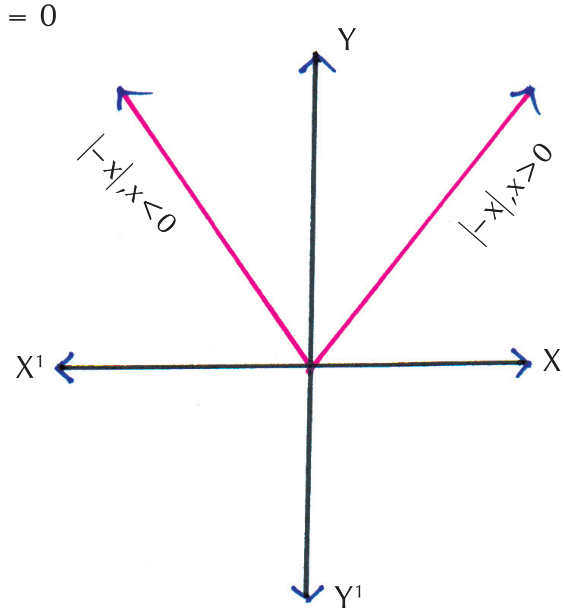
1. Take a geoboard of convenient size with given Co-ordinates are XOX' & YOY'
2. In the geoboard plot the st. line for $|x|$,

$$\begin{aligned} |x| &= x \text{ when } x > 0 \\ &= -x \text{ when } x < 0 \\ &= 0 \text{ when } x = 0 \end{aligned}$$

3. Take another geoboard with given co-ordinates XOX' and YOY' and plot the St. line for $|-x|$,

i.e. $$\begin{aligned} |-x| &= -x \text{ when } x < 0 \\ &= x \text{ when } x > 0 \\ &= 0 \text{ when } x = 0 \end{aligned}$$





Demonstration :

1. we see that in the geoboard, the graphical representation of $|x|$ in same with the graphical representation of $|-x|$.
2. So, we can say that $|x| = |-x|$

Activity - 34

Objective : To find the value of sine & cosine function in 2nd, 3rd & 4th quadrant using their values in 1st quadrant.

Learning Outcomes : From this activity student will be able to find out the values of Sinθ and Cosθ in four different quadrant.

Materials Required : A4 size paper, protractor, compus, scale, pencil, eraser

Preparation :

1. Draw two axis XOX´ & YOY´, where origin will be at the middle of the A4 size paper.
2. At the origin draw a circle with campus (i.e. origin will be the centre of the circle)
3. Draw a 30^o angle $\angle POX$ at the origin with the help of a protractor.

Demonstration :

1. The Co-ordinate of the point P is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ as $\angle POX = 30^o$
2. Now the Co-ordinate and the mirror image of P (where mirror is on OY) in the 2nd quadrant will be $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. As in the 2nd quadrant x is (-)ve.
3. Similarly the Co-ordinate of the mirror image of P´ [where mirror is on OX[1]] in the 3rd qudrant will be $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.
4. Also in the 4th quadrant the Co-ordinate of the mirror image of the point P (where OX is mirror) will be $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.
5. Therefore we see that as the Co-ordinate of point is $(\text{Cos}\theta, \text{Sin}\theta)$. The signs of cosine and sine in the four quadrants are given below.

	1st quadrant	2nd quadrant	3rd quadrant	4th quadrant
Sine-	+ve	-ve	-ve	+ve
Cosine-	+ve	+ve	-ve	-ve
Co-ordinate ofthe point P from the picture	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Activity - 35

Objective : To plot the graph of sinx, sin2x, 2sinx and sin $\frac{x}{2}$, using same Co-ordinate axes.

Learning Outcomes :

From this activity students will be able to compair graphs of a trigonometric function of multiples & submultiple angles.

Materials Required : Plyboard, graph paper, adhesive scale, coloured pencil, eraser.

Preparation :

1 Take a plywood and paste a thick graphpaper of convenient size on it.

2 Draw two mutually perpendicular lines on the graph paper and take them as Co-ordinate axes.

3 Prepare the table of order pairs for sinx, sin2x, 2sinx, sin $\frac{x}{2}$ for different values of x.

Demonstration :

Plot the order pairs (x,sinx), (x,sin2x), (x, sin $\frac{x}{2}$) and (x,2sinx) on the same Co-ordinate axes and joined the plotted ordered pairs by free hand curves in different colours as shown in the figure-1.

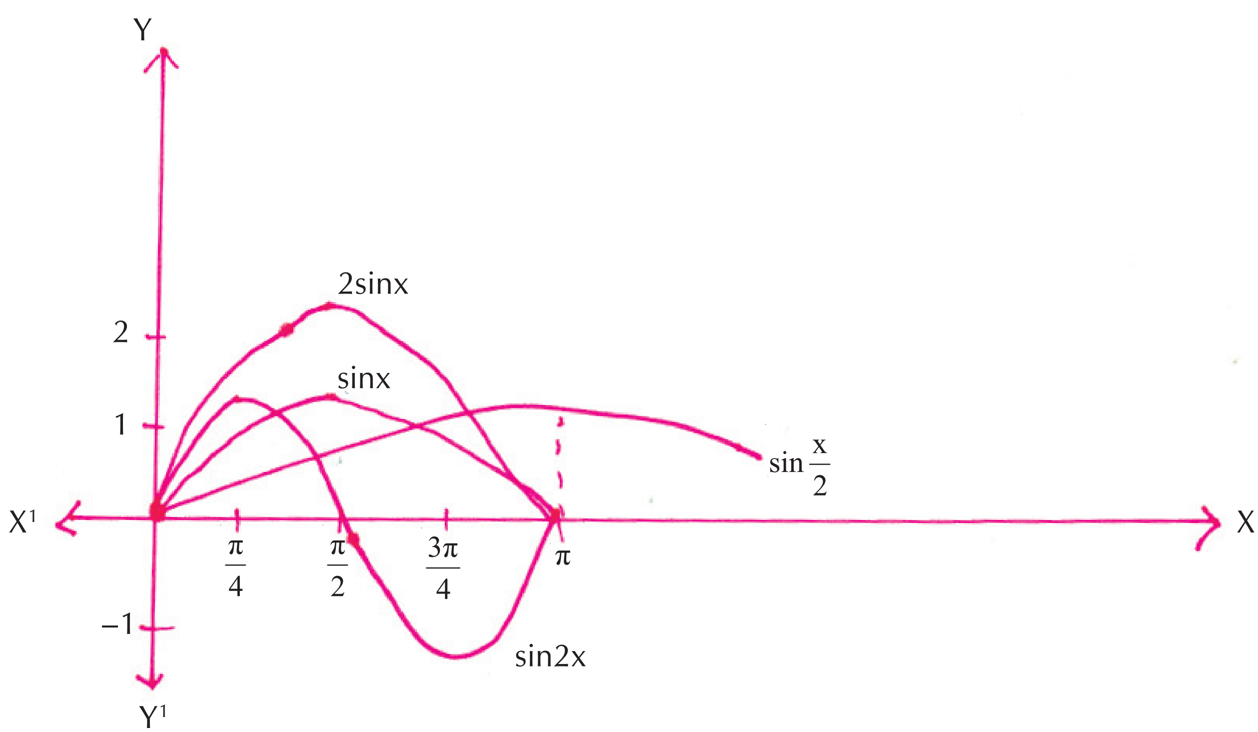


fig-1

Activity - 36

Objective : To find analytically $\lim f(x) = \dfrac{x^2 - c^2}{x - c}$

Learning Outcomes : From this activity student will be able to demonstrate the concept of $\lim\limits_{x \to c} f(x)$ when f(x) is not defind at $x = c$.

Materials Required : Pencil, Graph Paper, Calculator.

Preparation :

i) Consider the function f (x) is given by $f(x) = \dfrac{x^2 - 9}{x - 3}$ & this function is not defined at $x = 3$

ii) Take a graph paper of Convenient size.

iii) Draw two mutually Perpendicular lines on it & take them as Co-ordinate axes.

iv) Prepare the table of order Pair for $f(x) = \dfrac{x^2 - 9}{x - 3}$ when $x \to 3+$ and $x \to 3-$

Demonstration :

i) To find the order Pairs, take some values of x less than 3 & some other values of x more than 3

ii) In both cases, the values of x to be taken have to be very close to 3.

iii) Calculate the Corresponding values of f at each of the values of x taken close to 3.

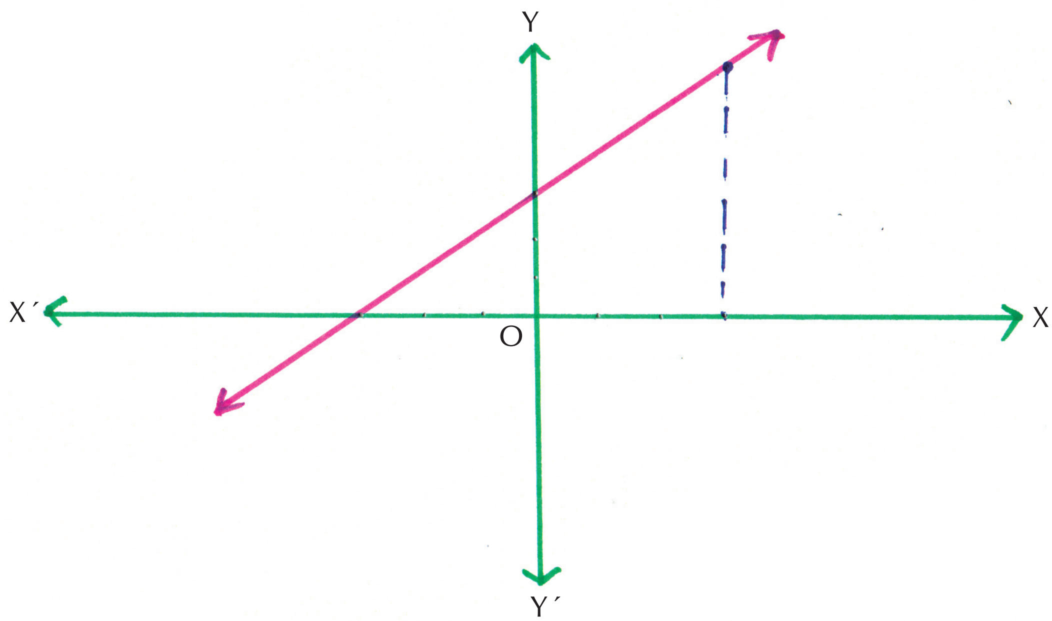
Table - 1

x	2.9	2.99	2.999	2.9999	2.99999	2.999999
f(x)	5.9	5.99	5.999	5.9999	5.99999	5.999999

Table - 2

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
f(x)	6.1	6.01	6.001	6.0001	6.00001	6.000001

iv) Plot the order Pair in the graph.



Observation :

1. When $x \to 3 + 0$ then f(x)
2. When $x \to 3 - 0$ then f(x)
3. $\lim_{x \to 3} f(x) =$ ____________________

Activity - 37

Objective : To interpret geometrically $\sqrt{-1} = i$ and its integral Power.

Learning Outcomes : From this activity students will be able to evalute any integral power of i.

Materials Required : Card board, Chart Paper, Sketch Pen, Compasses, ruler, adhesive, thread; nails,

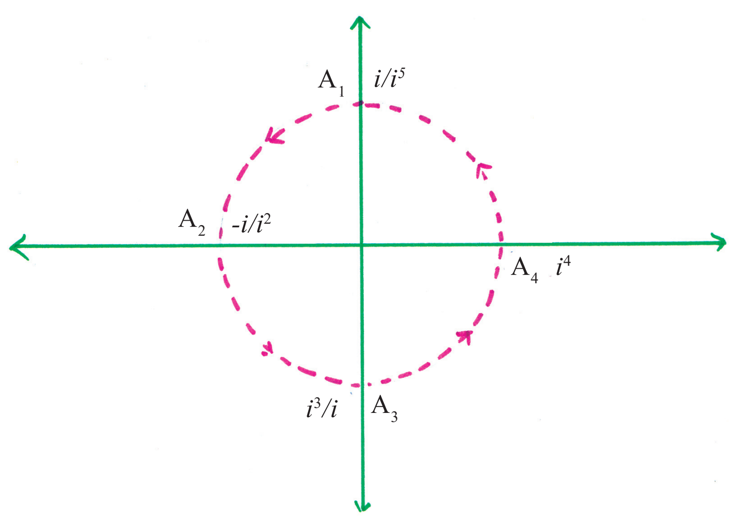
Preparation :

i) Paste a Chart Paper on the cardboard of a Convenient size.

ii) Draw Co-ordinates XOX' & YOY'.

iii) Take a thread of a unit length representing the numbers 1 along OX. Fix one end of the thread to the nail at O and the other end at A.

iv) Set free the other end of the thread at A and rotate the thread throngh angles of $90°, 180°, 270°$ and $360°$ and mark the free end of the thread in different cases as A_1, A_2, A_3, A_4 respectively.



Demonstration :

i) In the argand Plane, OA, OA_1, OA_2, OA_3, OA_4 represent 1,i,–1,–i,1 respectively.

ii) $OA_1 = i = 1 \times 1^o$

$OA_2 = -1 = i \times i = i^2$

$OA_3 = -i = i \times i \times i = i^3$

$OA_4 = 1 = i \times i \times i \times i = i^4$ and so on.

Each time rotation of OA by 90° is equivalent to multiplication by i. Thus i is referred to as the multiplying factor for a rotation of 90°.

Activity - 38

Objective : To find the domain of the following function $f(x) = \log_{\cos x} \sin x$

Learning Outcomes :

From this activity students will be able to understand the domain of logarithem function.

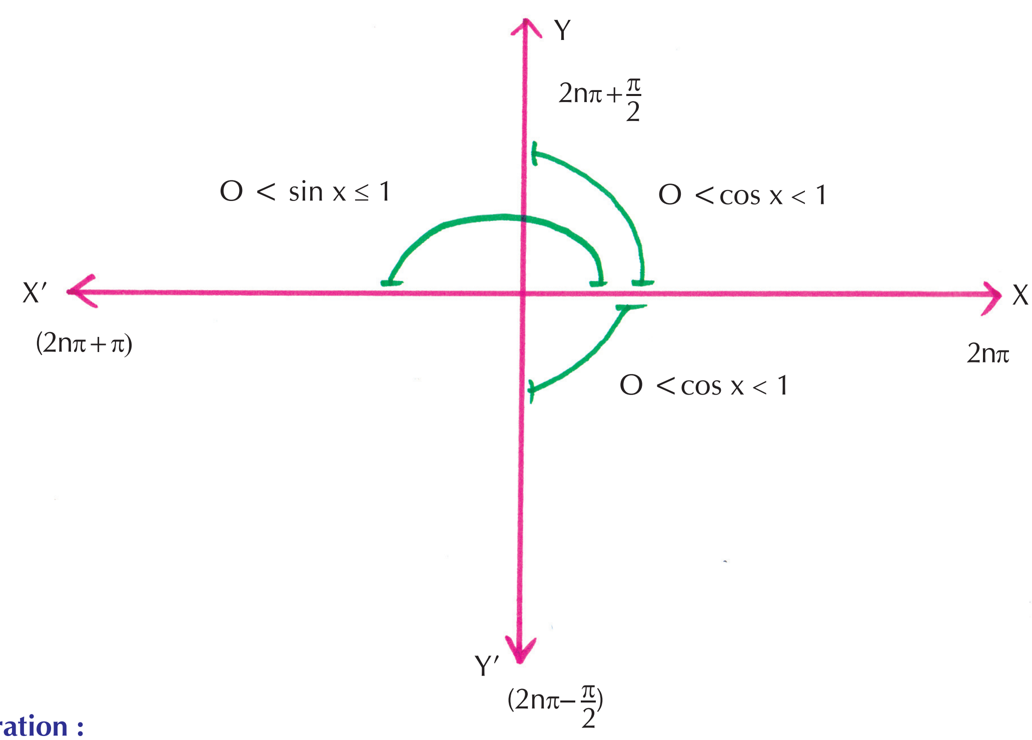
Materials Required : Plyboard, chart Paper, adhesive, ruler

Preparation :

i) Take a Plyboard and Paste a chart Paper of convenient size on it.

ii) Draw two mutually Perpendicular line on the chart Paper and take them as co-ordinate axes.

iii) Plot the function $f(x) = \log_{\cos x} \sin x$ on the co-ordinate axes where $f(x) = \log_{\cos x} \sin x$ is defined only when $0 < \sin x \leq 1$ & $0 < \cos x < 1$.



Demonstration :

i) From the above Picture, it can be said that the domain of f(x) in $2n\pi < x < 2n\pi + \frac{\pi}{2}$, $x < \mathbf{I}$,

[i.e, intessection of the sets $(2n\pi, 2n\pi + \pi)$ & $(2n\pi - \frac{\pi}{2})$ U $(2n\pi + \frac{\pi}{2})$]

Activity - 39

Objective : To Plot the graph of sin x, cos x, tan x, using same co-ordinate axes.

Learning Outcomes : From this activity students will be able to compair graphs of different trignometric functions.

Materials Required : Plyboard, graph Paper, adhesive, scale, coloured pencil eraser.

Preparation :

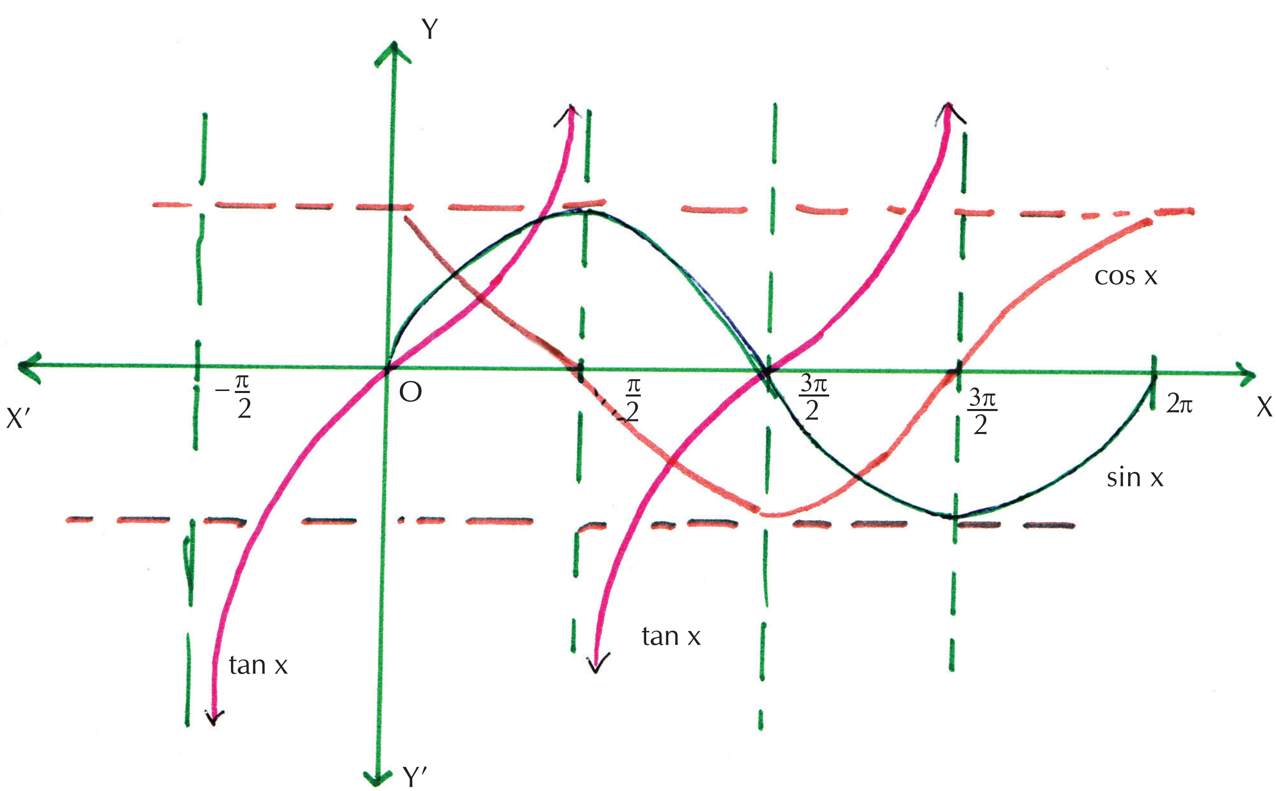
i) Take a plyboard and Paste a thick graphpaper of convenient size on it.

ii) Draw two mutually Perpendicular lines on the graphpaper and take them as co-ordinate axes.

iii) Prepare the table of order Pairs for sin x, cos x, tan x for different values of x.

Demonstration :

Plot the order Pairs (x, sinx), (x,cosx) , & (x, tanx) on the same co-ordinate axes and joined the ploted orderpairs by free hand curves in different colours as shown in the figure.



Activity - 40

Objective : To establish the meaning of $x \to a$.

Learning Outcomes : After this activity the learner will be able to understand the meaning of $x \to a$.

Materials Required : white paper, pencil, eraser.

Preparation :

i) Draw a real line OX.

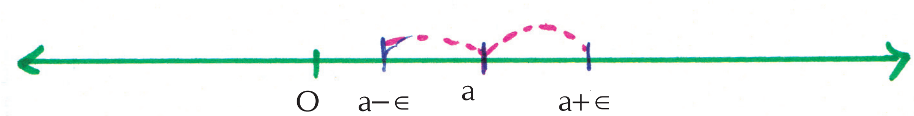
ii) Consider a particular point $x = a$ on OX.

iii) Let $\in$ be a pre asign positive number however small.

iv) Consider two pts $a-\in$ & $a+\in$ as shown below.

Demonstration :

$x \to a \Rightarrow 0 < |x-a| < \in$, is a pre asigngd possitive no. however small, $\Rightarrow a - \in < x < a + q$ ie x is greater than a - $\in$ & less than a + $\in$



Activity - 41

Objective : To understand geometrical interpretation of scalar triple product of three vectors.

Learning Outcomes : Students will be able to explain the scalar triple product of three vectors geometrically.

Materials Required : Wires, soldering wire, wire cutter.

Preparation :

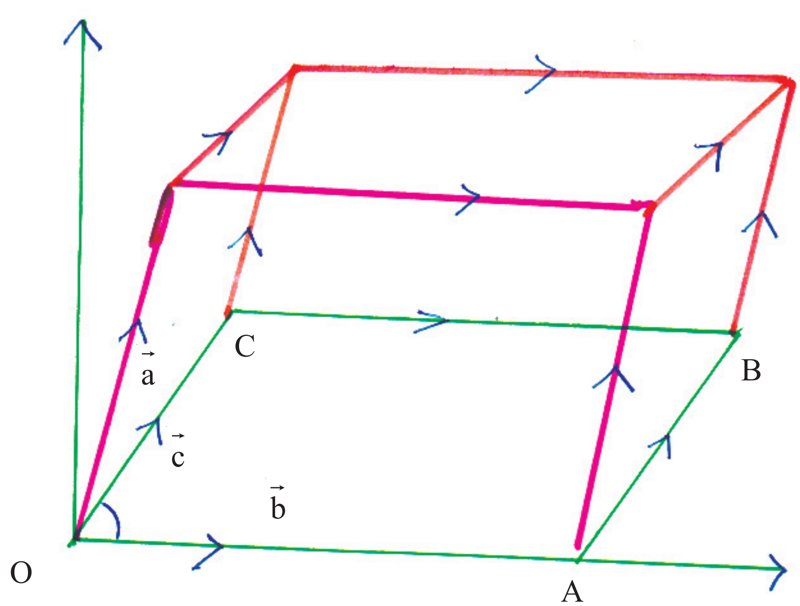
i) Cut four wires of about 15 cm length to show length of $\vec{a}$.

ii) Cut four wires of about 20 cm length to show length of $\vec{b}$.

iii) Cut four wires of about 10 cm length to show length of $\vec{c}$.

iv) Construct a parallelopiped with the heep of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

v) Fix a wire perpendicular to the bare parallelogram to show of $\vec{b} \times \vec{c}$.



Demonstration :

(i) $\vec{b}$ and $\vec{c}$ represents adjacent sides of the base of the parallelogram OABC.

(ii) The line perpendicular to the base of the parallelogram is along $\vec{b} \times \vec{c}$.

(iii) Projections of $\vec{a}$ along $\vec{b} \times \vec{c}$ corresponds to the height of the parallelopiped.

(iv) $\vec{a} \cdot (\vec{b} \times \vec{c})$ represents volume of the paralleopiped.

Observation :

(i) where $|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}| \sin\theta$ where be the angle between $\vec{b}$ & $\vec{c}$

(ii) $|\vec{c}| \sin\theta =$ perpendicular length from C to OA.

(iii) $|\vec{b} \times \vec{c}|$ represents area of the parallelogram OABC.

(iv) $\vec{a}.(\vec{b} \times \vec{c})$ represents area of the base × height of the parallelopiped i.e. volume of the parallelopiped.

Activity - 42

Objective : To understand the concept of probability.

Learning Outcomes : Students will be able to understand the idea of sample space, events and their probability.

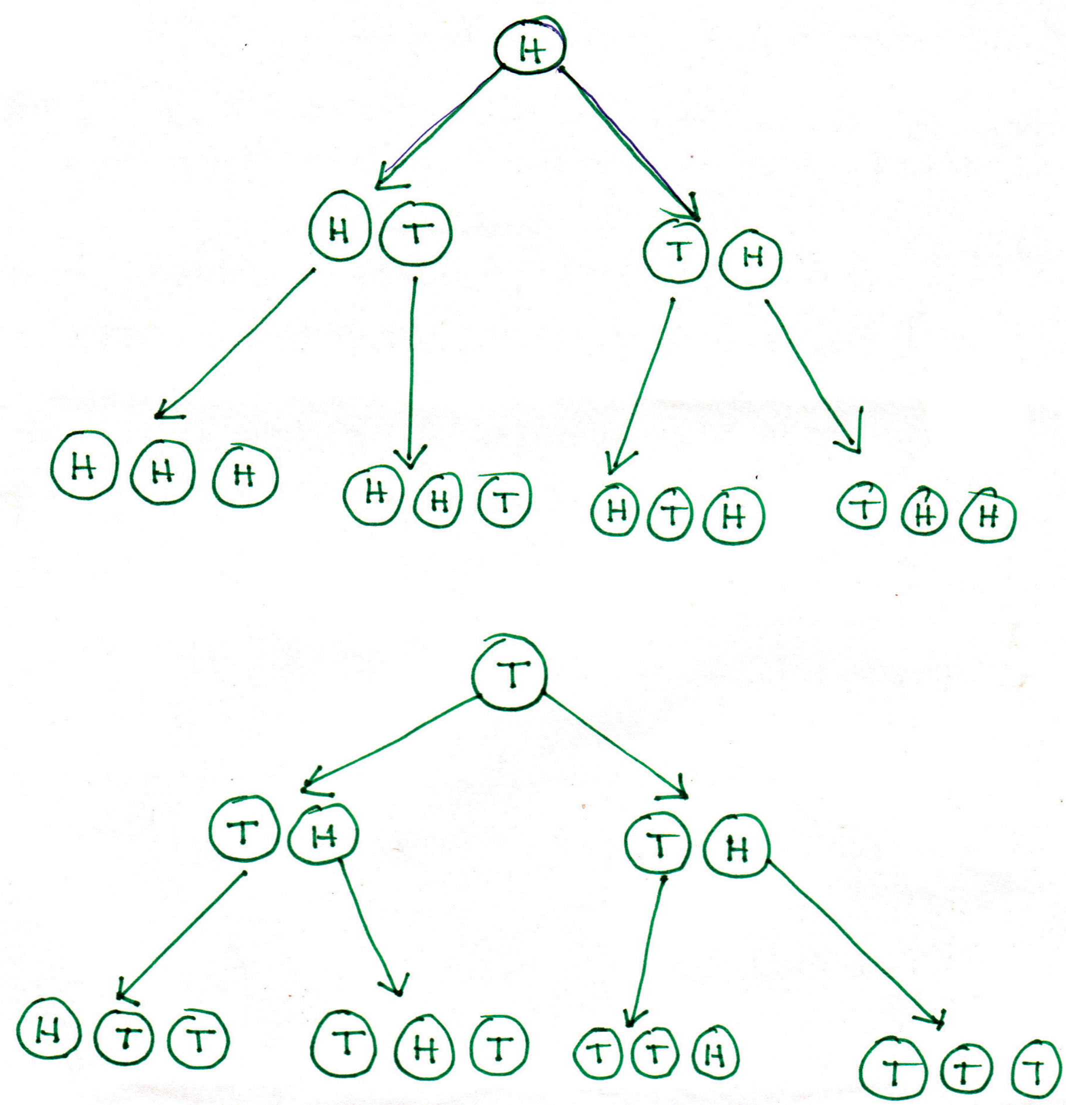
Materials Required : Handboard, Chart paper, metal dics, marker, adhesive.

Preparation :

i) Take a piece of hardboard and cover by chart paper.

ii) Fix metal discs on the chart paper for showing different sample points in differnt events.

iii) Take a marker and write 'H' and 'T' on the discs where 'H' stands Head and 'T' stands 'Tail'.



Demonstration :

Let S denote the sample space. Then S can be demonstrated in the following manners.

(i) If a disc is thrown once, the sample space S={H, T}.

(ii) If a disc is thrown twice, the sample space S={HT, TH, HH, TT}

(iii) If a disc is thrown thrice, the sample space S={HHH, HTT, THT, TTH, HHT, HTH, THH, TTT}

(iv) Take A as a subset of S such that it has at least one head then. A={HTT, THT, TTH, HHT, HTH, THH, HHH}

(v) Take B as a subset of S such that it has at least two tail. then B={HTT, THT, TTH, TTT}

Observations :

(i) The possible number of outcomes when a disc is thrown once, twice, trice are ____________.

(ii) The possible number of outcomes when a disc is thrown thrice and getting at least one head is ____________.

(iii) Probability of an event A is ____________ .

(iv) The possible number of outcomes when a disc is thrown thrice and getting at least two tail is ____________.

(v) Probability of an event B is ____________ .

Project on Mathematics

What is mathematics Project?

Mathematics project helps students to understand a specific mathematical concept or idea. When student are making mathematics project he / she is doing an in-depth study of the topic. Mathematics project can be done about any type of math concept from Elementary to Higher Secondary. Focus on the topic for which students are doing to do a math concept. It is important that student have a full understanding of the concept. First student find a topic either research based or concept developing, then take advice from the guide teacher before finalising your project topic.

How do write Mathematics Project.

The following proforma we may follow to make Mathematics Project :

1. Name of the Project (Title of the project).
2. Introduction (Overview)
3. Objective of the project.
4. Methodology
5. Experimental data (If the project is research based study)
6. Analysis
7. Conclusion/Inference
8. Limitations of the study (If the project is study based research)
9. References
10. Appendix/Questionnaire

Date : Signature of the student

Signature of the teacher

Project - 1

Title : Project on history, development and application of Complex Numbers, Contribution of Mathematicians in the relevant fields with a historical approach.

Introduction : We all know that mathematics is the language of Science, that is, without mathematics the evolution of the origin of science is not possible. It is all known that mathematics was practiced mainly in Greece, India and Arab before many years before the birth of Christ. All the theories and formulas applied in mathematics are not invent in a day. It is said that the time was a few days before the second world war many mathematician in the mathematics department like other department were present at University in Germany. A famous mathematician thinking about a hard theory but we did not solve the problem. One day he sat in canteen and doing maths on the table. Long time spend for doing that but he did not solve the problem and the solution was not coming. He left home, leaving work unfinished. But surprisingly the next day he urned to the table and saw an unknown mathematician solve this hard problem. There are numerous myths behind every mathematical theory.

Objective : We know that rational number and irrational number belong to real number. One of the main characteristic of all real number is that its square is always positive. So, square of these $1, \frac{1}{2}, \sqrt{2}, \frac{\sqrt{5}}{2}$ real numbers are $1, \frac{1}{4}, 2, \frac{5}{4}$ and all are positive. For these reason if any number, square is negative, it is not called number. Square of $\sqrt{-1}$ and $\sqrt{-3}$ are –1, – 3 that means negative. $x^2 + 1 = 0, x^2 + 3 = 0$, these types of numbers arises only when we are solving these types of equations. So, these types of number which are not real number, are called imaginary number or complex number. Complex Number takes an important role in numberology. So, the main purpose of this number are to acquire knowledge and the history of these number, history of origin and the application in mathematics.

Procedure and Data Analysis : According to subject teacher content, data and mathematical problems of complex number were being collected from different types of books and websites.

Story of Origin of Complex Number : Many people are surprised to find that the ideas of complex number has arisen primarily for solving the quadratic equation, never to solve trinity equation.

The following informations are bound in the explanation of the history of the origin of this complex number.–

(i) Al–Khawarizmi, in his algebraic book solved a variety types of quadratic equations that are in perfect alignment with the solutions carrently taught by the school. Although the solution is limited to the positive number and its proof was based on a geometric basis. It is assumed that this concept is at the core of Greek and Hindu mathematics. The algebric texts of these mathematics, which were known to Arabs were translated into Latin by Gerard of Cremona (1114 - 1187), first introduced in Italy and then expanded by the hand of Fibonacci. In the year 1225, Leonardo was introduced to the emperor Frederick II as the perfect solution to any complex mathematical problem. And a local mathematician kept several problems in front of him, one of them is $x^3 + 2x^2 + 10x - 20$.

(ii) Scipione del Ferro (Until 1526 he taught at the University of Bolona). He was the first to solve the equation. The source of this solution was told to dear student Antonio Mario Fiore. Then in a competition, Fiore called mathematician Tartagilla. Tartagilla was able to redesign the formula before the competition and won the contest. Tartagilla again told Gerolamo Cardano on the condition of his commitment to protect the sourced privacy. Cardano was able to prove the formula by applying his own knowledge and ideas. Later, Cardano learned the source was once with del Ferro. He met with del Ferro's relatives and verified his authenticity with del ferro's formula and solution. Cardano subsequently published the book Ars Magna (1545). On the application of the formula in all three cases, it may be said that Cardano did not refuse to mention del Ferro and Tartagilla as the first inventors of the formula.

Cardano first introduced the use at algebraic complex number $a+\sqrt{-b}$. But there were some confusion. There mentioned a complex problm in Ars Magna. "To divide 10 into two parts, the product of which is 40."

Naturally, the solution can be thought of as dividing 10 into two equal are 5. Its square is 25, 25 to 40 minus is –15, if its square root is adding with 5 or subtracting from 5, the multiplication of the parts is 40. {25 – (– 15) = 40}

Rafael Bombelli first use $\sqrt{-1}$ – this symbol in his 'I' Algebra (1572 and 1579) book. He followed the path of Cardano in the case of trident equation. However, he had extensive discussions about casus irreducibillis.

The equation given by Bombelli is

$x^3 = 15x + 4$

The application of Cardano's formula can be written :

$$x=\sqrt[3]{2+\sqrt{-121}}+\sqrt[3]{2-\sqrt{-121}}$$

Bombelli noticed that x = 4 is a solution to the triplicate equation. He wrote x = 4 to give the expression given in Cardano's formula as follows :

$$\sqrt[3]{2+\sqrt{-121}}=a+bi$$

$$and\sqrt[3]{2-\sqrt{-121}}=a-bi$$

Available through algebraic manipulation

a = 2 and b = 1

So, x = a + bi + a + (–bi) = 4

His memorable comment is :

"At first, the thing seemed to me to be based more on sophism then on truth, but I searched until I found the proof."

Rene Descartes (1596 - 1650) solved the problem of geometric by applying algebra in his La. Geometric. He shows a connection of geometric impossibility with the imaginary number.

Descartes's comment on imaginary number is :

"Far away equation one can imagine as many roots (as its degree would suggest), but in many cases no quantity exists which corresponds to what one imagines."

Abraham de Moivre (1667 - 1754) left France and moved to London at the age of 18. There he became friends with Newton. In 1698, he stated that Newton knew about his inventions long ago which are famous in the name of 'de Moivre's Theorem'

His theorem is :

If n is positive whole number or negetive whole number, then, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ and if n is a rational number then $\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$

This formula was used by Newton at cube root of Cardano's formula.

John Wallis (1616 - 1703) stated in his 'Algebra' entitled book that negative sum, in which the mathematician is hesitant. Such as denotes the positive and negative number signs to the right of the zero point along a straight line. He gave a little advanced geometric explanation of $\sqrt{-1}$.

L. Euler (1707 - 1783) uses $i = \sqrt{-1}$ and showed complex number as coordinate point. But he could not establish a satisfactory basis for the complex number.

The formula which is used by Euler is :

$x + iy = r(\cos\theta + i\sin\theta)$

and he shows $z^n = 1$ as the vertex of a rectangular polygon. Moreover, he defines complex indices and $e^{i\theta} = \cos\theta + i\sin\theta$ prove this difference.

Norwegian mathematician Casper Wessel (1745 - 1818)

Understood the concept perfectly and presented it in a perfect way. In 1797 Wessel submitted his essay" On the Analytic Representation of Direction : An Attempt" to the Royal Danish Academy of Sciences which was published in the Academy's 1799 memories. Written in Danish, this article remained unnoticed to all until 1897. Unless it were in the hands of Denmark's famous mathematician, Sophus Christian Jnel, we would not have known that Vector Algebra was created from Wessel's thought.

William Rowan Hamilton (1805 - 1865) defines the ordered pairs of real numbers in an article published in 1831. He also provided the algebraic idea of addition and multiplication of complex number. He said that complex number is the ordered pair (a,b) of real number, where a and b are real number. This ordered pairs follow the rituals as follows :

(i) $(a, b) \pm (c, d) = (a \pm c, b \pm d)$

(ii) $(a, b) = (c, d) \Leftrightarrow \begin{matrix} a = c \\ b = d \end{matrix}$

(iii) $(a, b)(c, d) = (ac - bd, ad + bc)$

(iv) $(a, b) \div (c, d) = \left(\dfrac{ac+bd}{c^2+d^2}, \dfrac{bc-ad}{c^2+d^2}\right)$

$(\text{where } c^2 + d^2 \neq 0)$

Jean Robert Argand (1768 - 1822) was a businessman in a Parisian book. It is not known whether he had any mathematical education. In 1806 Argand published a little book. He did not write his name in his book? The title of the book is " Essay on the Geometrical Interpretation of Imaginary Quantities". It is our fortune that a copy of the booklet falls into the hands of a mathematician named A. Legend (1752 - 1833) which he again mentions in a letter to the renowned mathematician Francois Francais, and send the copy to him. When Francois died, all his writtings and collections were passed on to his younger brother Jaques.

Carl Fried Gauss (1777 - 1855) had a geometric interpretation of the complex number from 1796, but remained unpublished until 1831. Because at that time he made his statement in public through the Royal Society of Gottingen.

" If this subject has hitherto been considered from the wrong viewpoint and thus enveloped in mystry and surrounded by darkness, it is largely an unsuitable terminology which should be blamed. Had $+1$, -1, and $\sqrt{-1}$, instead of being called positive, negative and imaginary unity been given the names say of direct inverse and lateral unity, there would hardly have been any scope for such obscurity.

Details on the development of complex number :-

Any ordered pairs (a,b) where a and b is a real number is called a complex number. For example-$a+ib$, $i = \sqrt{-1}$ (iota), here a is real number and b is imaginary part of complex number. If the coefficient of i is zero then the zodiac is purely real and if the real part of the zodiac is zero, the zodiac is purely imaginary. The following is a discussion of addition, subtraction, multiplication and division of complex number. This is called Algebra of complex numbers.

(i) if $a+ib$, $c+id$ are two complex numbers then

$$(a+ib)\pm(c+id)\pm(a\pm c)+i(b\pm d)$$
$$(ii) a+ib=c+id \langle=\rangle a=c, b=d$$
$$(iii)(a+ib)(c+id)=ac+aid+ibc+i^2bd$$
$$=(ac-bd)+i(ad+bc)$$
$$[i^2=-1]$$
$$(iv)\ (a+ib)\div(c+id)=\frac{(a+ib)(c-id)}{(c+id)(c-id)}$$
$$=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}$$

Where the permissive complex number of $c+id$ is $c-id$, that is, two complex zones with equal parts if only the imaginary part of the symbol is opposite to each other or it can be said that conjugate of $a+ib$ of a complex number is $a-ib$. If exists so that the multiplication of two number is purely real. These two number are known as relative on complementary complex number to each other. This complementary complex number is denoted by $\bar{z} = a+bi$.

Polar Coordinates : Suppose $z = 1+i$ is a complex number that is drawn in Fig. The position of a in cartesian form is (1, 1) coordinates or the length of $\overline{OA}$ and be guided by direction.

According to Pythagora's theorem :

$\overline{OA} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore |Z| = r = \sqrt{2}$

The angle which is produced by OA in the positive side of the real axis is

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^0$

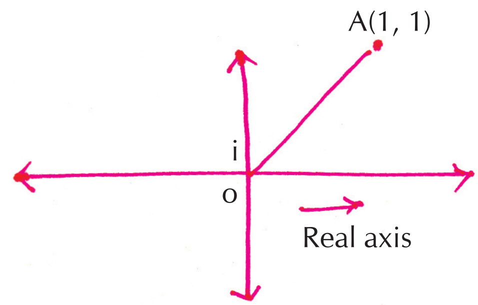


figure-1

It is written in arg (z) = 45^0. It is said that argument $|Z|$ and arg Z, parameters are to be equivalent to polar coordinates. The simple relation of polar and cartesian is

$a = r\cos\theta \; , b = r\sin\theta$

$\therefore z = a + ib = r(\cos\theta \; + i\sin\theta)$

Where $|Z| = r$ and $\arg(Z) = \theta$

If the value of θ is between $-\pi$ and π that is to say if $-\pi < \theta \leqslant \pi$ then the argument is calld principal argument. We can see that knowing the modulus and arguments of the complex number means knowing the position of the number. So therefore the formula for determining main argument through the different position of complex number is –

(1) If complex number is in first quadrant then $\theta = \tan^{-1}\left|\frac{b}{a}\right|$ (where z = a + ib)

(2) If complex number is in second quadrant then $\theta = \pi - \tan^{-1}\left|\frac{b}{a}\right|$

(3) If complex number is in third quadrant then $\theta = -\pi + \tan^{-1}\left|\frac{b}{a}\right|$

(4) If complex number is in forth quadrant then $\theta = -\tan^{-1}\left|\frac{b}{a}\right|$

The properties of modulus and argument :-

Suppos z_1 & z_2 are two complex number

(i) $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

(ii) $|Z_1 - Z_2| > \big| |Z_1| - |Z_2| \big|$

(iii) $|Z_1 Z_2| = |Z_1| \ |Z_2|$

(iv) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

(v) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

(vi) $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

If we assume $z_1 = r_1 \cos\theta_1 + ir_1 \sin\theta_1$ and $z_2 = r_2\cos\theta_2 + ir_2 \sin\theta_2$ then $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$
Beside the properties of permissive complex number are :

(i) $(\overline{\overline{z}}) = z$

(ii) $\overline{z_1 \mp z_2} = \overline{z_1} \pm \overline{z_2}$

(iii) $\overline{z_1 z_2} = (\overline{z_1}).(\overline{z_2})$

(iv) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

These properties are used to solve different mathematical problem of complex number. We have already discussed the important theory of the complex number of De Moivre. Here are some examples that have used this theory.

Example - 1. $\frac{(\cos\theta + i\sin\theta)^5}{(\cos\theta - i\sin\theta)^4}$ Find the simplest form.

Solution : $(\cos\theta + i \sin\theta)^5 (\cos\theta - i \sin\theta)^{-4}$

$= (\cos 5\theta + i \sin 5\theta)(\cos(-4)\theta - i \sin(-4)\theta)$

$= (\cos 5\theta + i \sin 5\theta)(\cos 4\theta + i \sin 4\theta)$

$= \cos(5\theta + 4\theta) + i \sin(5\theta + 4\theta)$

$= \cos 9\theta + i \sin 9\theta$

Example - 2 : $(1)^{\frac{1}{3}}$ – Find the value ?

Solution : $1^{\frac{1}{3}} = (\cos 0^\circ + i \sin 0^\circ)^{\frac{1}{3}}$

$= \{\cos(2k\pi + 0) + i \sin(2k\pi + 0)\}^{\frac{1}{3}}$

$= \cos\frac{2k\pi}{3} + i \sin\frac{2k\pi}{3}, \ k = 0,1,2$

∴ If k = 0 then the value is 1

k = 1 then the value is $-\frac{1}{2}+\frac{i\sqrt{3}}{2}$

k = 2 then the value is $-\frac{1}{2}-\frac{i\sqrt{3}}{2}$

That is, the application of this theory to express simplified forms of large complex numbers and determining any root of any number is undeniable.

Euler's Theorem :

$$e^x = 1 + x + \frac{x^2}{21} + \frac{x^3}{31} + ...$$

We know that

$$\cos x = 1 - \frac{x^2}{21} + \frac{x^4}{41} - ......$$

$$\sin x = x - \frac{x^3}{31} + \frac{x^5}{51} - ......$$

e^{ix} = cosx + i sin x, this is the law of Euler's formula.

Loci in the complex plane :

If a complex number z is changed such that $|z-1| = 3$ then it can be written

$\sqrt{(x-1)^2 + y^2} = 3$ (where z = x + iy)

$\Rightarrow (x-1)^2 + y^2 = 9$

Obviously, in cartesian geometry, this is an equation of a circle whose center coordinate of the centre is (1, 0) and radius is 2. Now the coordinates of the center will be 1 + 0i

This method is acceptable for solving any problem.

The following information should be kept in mind to solve different problems of complex number.

(i) If z_1 and z_2 are two complex number then the distance between z_1 and z_2 is $|z_1 - z_2|$

(ii) If two points $A(z_1)$ and $B(z_2)$ are divided by one point $P(z_1)$ in the ratio $m_1 : m_2$ then

$z = \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2}$, m_1 and m_2 both are real number.

(iii) The equation of the straight line joining the point z_1 and z_2 is $(z-z_1)(\bar{z}-\bar{z}_1)=(\bar{z}-\bar{z}_1)(z-z_1)$

(iv) If the point z_1, z_2, z_3 are one line then $(z_1-z_2)(\bar{z}_1-\bar{z}_3)=(\bar{z}_1-\bar{z}_2)(z_1-z_3)$. These opposite is also applicable.

(v) i ≠ 0, > 0, < 0

(vi) amp z – amp (–z) = $\pm\pi$ when amz > 0 or amz < 0

(vii) If the three point z_1, z_2, z_3 are the vertex of an equilateral triangle then

$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$ or, $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$ or, if $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$ then the three points z_1, z_2, z_3 will be the vertex of an equilateral triangle.

(viii) If $\left|z + \frac{1}{z}\right| = a$ then the largest and smallest values will be $\frac{a + \sqrt{a^2 + 4}}{2}$ and $\frac{-a + \sqrt{a^2 + 4}}{2}$

(ix) The equation of any straight line in imaginary surface is $\bar{a}z + a\bar{z}$ = real number.

(x) $|z - z_0| = r$, this equation is the equation of a circle whose center is z_0 and radius is r and $|z - z_0| < r$ is the center of the circle. $|z - z_0| = r$ is the upper part and $|z - z_0| > r$ is the outer part of the circle.

(xi) $z\bar{z} + a\bar{z} + \bar{a}z + k = 0$ (k, real) indicates the equation of circle who center is –a and radius $\sqrt{|a|^2 - k}$ unit.

(xii) If $|z - z_1| + |z - z_2| = 2a$ where $2a > |z_1 - z_2|$ then the path of z will be a ellipse where focii are z_1 and z_2 and $a \in R^+$

(xiii) If $|z - z_1| - |z - z_2| = 2a$ where $2a < |z_1 - z_2|$ then the path of z will be supremacy whose focii are z_1 and z_2 and $a \in R^+$

(xiv) If $\left|\frac{z - z_1}{z - z_2}\right| = k$ and $k \neq 1$ then we get a equation of circle, or If $k = 1$ then we get a equation of straight line.

(xv) If arg $\frac{(z_2 - z_3)(z_1 - z_4)}{(z_1 - z_3)(z_2 - z_4)} = \pm\pi, 0$ then the point z_1, z_2, z_3, z_4 will lie in one circle.

The practical application of complex numbers :

There are complex number applications in many subject of science like signal proccessing, control theory, electromagnetism, Fluid dynamics, quantum mechanics, analysis etc. The details are discussed below :

The concept of complex number are used in signal analysis. The maximum value of a particular sin frequency is $|z|$ and period is amp z.

Control theory also converts time-domain to frequency region via laplace transform. Also, in the equation of dynamics and electromagnetic theory, electrical engnineering are used with the help of complex number.

Conclusion :-

The history of invention and application of complex number has made it possible to gain many knowledge. In this way, other areas of mathematics can be gain knowledge, this will increase students' interest in mathematics.

Acknowledgement :

In this section, students will be grateful to those who have contributed to the implementation of the entire project.

Project - 2

Title : Project on properties and application of Parabola and Ellipse.

Introduction: The branch of mathematics in which algebra is applied to solve geometrical problems is called Analytical Geometry or Co-ordinate Geometry. In this case, the co-ordinate axes may be inclined at rightangle or not. If the co-ordinate axes are inclined at right-angle, then the co-ordinates are called rectangular co-ordinates and if the co-ordinate axes are not inclined at right angle, then the co-ordinates are called oblique co-ordinates.

The famous French philosopher and Mathematician Rene Descartes introduced these two type of co-ordinates According to his name these two co ordinates are called cartesian co-ordinates.

Conic section is the most important Part of Co-ordinate Geometry. This name originates in a vertical cone with several intersections.

Rene Descartes Published his famous book 'La Geometry'. In 17th Century French Mathematician Fermal also pioneered the development of analytical geometry. Descartes made a connection between Algebra and Geometry by the position of any point on a plane.

Objective: The most important area of co-ordinate geometry involves the concepts of conic sections. Co-ordinate Geometry is widely used in Atomic Physics, Space Science, Applied Science Structural Engineering. From this project a deep knowledge on the concept about Parabola and Ellipse and their usages and applications in different branches of mathematics can be strengthen.

Data collection and Analysis: If a right circular cone is differently intersected by a Plane then the different intersections are called conic section. If the plane in the side figure is Parallel to the base of the cone or Perpendicular to the axis, then the intersection will be a circle. If the intersection plane is diagonal rather than Parallel to the base and intersections all the generating lines of the cone, the intersection will be an Ellipse.If the intersection plane is parallel to one and only one generating line of the cone. the intersection is Parabola. If the Plan is Parallel to the axis of revolution (the axis) then two conic section is a hyperbola.

Each of these has one common Property; Each curve is the locus of a point moving in a Plane at a fixed point that is Proportional to its.distance from a fixed line (not passes through that fixed point) at any given point in that plane. The specific point in the plane is the focus of the cone and the specific line within the plane is called the directrix of the cone. This Property is called the property of focus directrix. The ratio of the distance betwen a fixed point within a plane to the moving point and the distance from a fixed straight line to a point is called the ecentricity of the conic. It is denoted by e. In the property of focus-directrix, the absolute value of distances is always taken. Because of this, ecentricity is always positive. If $e = 1$, than the locus is parabolical and the conic is called parabola. Again if $e > 1$ then the locus is Hyperbolical and the conic is called hypenbola and if $e < 1$ then the locus is elliptical and the conic is called ellipse.

Parabola: Since $e = 1$, so, if a moving Point in the plane is equidistant from a Fixed point (focus) and a fixed line (directix) (not through the fixed point), then the locus of the moving point is called parabola. that fixed point is called focus and the fixed line is called the directix of the parabola. The straight line passing through the focus and perpendicular to the directrix is called the axis of the parabola. In which point the axis intersect the parabola is called the vertex of the parabola.

Let S be a fixed point (focus) and l_1, l_2 be the fixed straight line (Direction). Let $\overrightarrow{AX}$ (x axis) and $\overrightarrow{AY}$ (y axis) be two axes through A as vertex.

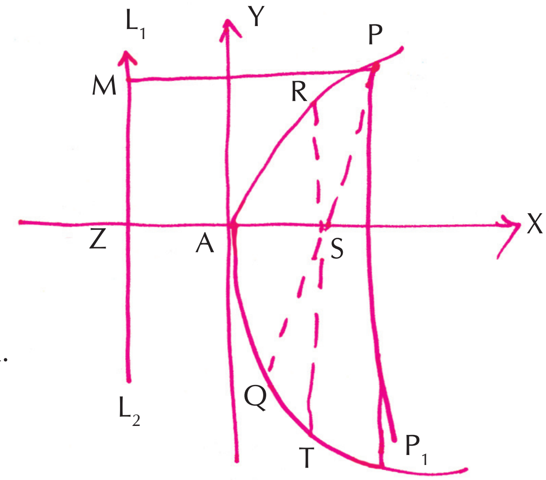
Let $\overline{ZA} = \overline{AS}$ = a unit

If P(x,y) be any point then SP = PM

or, $(x-a)^2 + y^2 = \overline{ZN}^2 = (a+x)^{22}$

or, y^2 = 4ax, this is the general equation of the parabola.

i.e. PN^2 = 4.AS.AN



$\overline{PQ}$ is a chord of the parabola. which chord is passing through focus and perpendicular to the axis is called the latus recturn. i.e. 4a is the length of the latus recturn.

Important informations about different types of parabolas are given bellow:–

(i) Equation of Parabola: y^2 = 4ax Co-ordinate of vertex: A(o,o), Equation of the axis: y = o ($\overrightarrow{AX}$ axis), Co-ordinate of focus: S (a,o), Length of the latus recturn: (RT) = 4a, Equation of the directrix: $L_1\ L_2$ = x + a = 0

(ii) Equation of the parabola: y^2 = -4ax Co-ordinate of vertex: A(0,0), Equation of the axis: y = 0 ($\overrightarrow{AX'}$), Co-ordinate of focus: S (a,o), Equation of directrix: x = a ($\overleftrightarrow{L_1L_2}$), Length of the latus rectum: (RT) = 4a.

(iii) Equation of the Parabola: x^2 = 4ay, Co-ordinate of vertex: A(o,o), Co-ordinate of focus: S(o,a), Equation of the axis ($\overrightarrow{AX}$): x = 0, Equation of the directrix (L_1L_2): y = -a, Length of the latus rectum: (RT) = 4a.

(iv) Equation of the parabola: x^2 = -4ay Co-ordinate of vertex: A(o,o), Co-ordinate of focus: S(o,-a), Equation of the directrix ($\overleftrightarrow{L_1L_2}$): y = a, Length of the latus rectum = 4a.

(v) Equation of the parabola: $(y-\beta)^2$ = 4a $(x-\alpha)$, Co-ordinate of vertex: A (α,β), Co-ordinate of focus: S($a+\alpha,\ \beta$) Equation of the axis $(\overrightarrow{AX})$: y = β, Equation of the directix ($\overleftrightarrow{L_1L_2}$): x- α + a = o, Length of the latus rectum = 4a.

Similar to (v), if the vertex of the Parabola is on any quadrant and the axis of the Parabola is Parallel to the original axis than get the equation of the parabola is given below:

$(y-\beta)^2 = -4a(x-\alpha)$

$(x-\alpha)^2 = 4a(y-\beta)$ and $(x-\alpha)^2 = -4a(y-\beta)$

Properties of Parabola:

1. If tangent and normal are drawn from any point P on the Parabola and they meet the axis of the parabola at T_1 and N respectively then, $\overline{ST_1} = \overline{SN} = SP$ and $\angle PT_1S = \angle SPT_1$
2. If the tangent drawn at the point P meets the direcfix at K then the angle $\angle KSP$ will be right angle.
3. Tangents drawn from the edges of the focal chord of the Parabola intersect the directrix at right angle.
4. If SQ be the Parpendicular to the tangent drawn at P then Q lies on the tangent drawn at vertex and $\overline{SQ}^2 = \overline{AS}.\overline{SP}$
5. PP_1 is the double ordinate [i,e PP_1 is a perpendicular chord so that the length of $\overline{AV}$ is the half of that chord PP_1

   i,e. $\overline{AV} = \overline{PV} = \overline{VP_1}$

   Here $= \angle PAP_1 = 90°$ i,e edges of double ordinate inclined to the vertex of the Parabola at right angle.
6. $x = at^2$ and $y = 2at$, where t is the Parameter, is the Parametric equation of the Parabola.

Practical application of parabola:-

In the case of reflection of light, the light that enters it travelling parallel to the axes of symmetry of parabola and reflected to the focus. Conversely light that orginated a point source at the focus is reflected into a parallel became leaving the parabola parallel to the axis of symmetry. The same effects occur with sound and other waves. This reflective property is the basis of many practical uses.of parabola. The parabola has many important practical applications, that may be in physics or engineering or any other branches of science. For example parabolic antenna, parabolic microphone, to automobile headlight reflectors and the design of ballistic missiles etc. Further the diagrams of the laws of average braking distance and average stopping distance of motor vehicles are parabolical. Here only one condition is cars having brakes in first class condition and moving on dry level hand surfaces. The reflective property of parabola is used in water heater. The heat source is at the focus of the parabola and heat is concentrated parabola and heat is concentrated in Parallel rays. Satellite Dishes work on this same principle. Incoming waves are concentrated to the focus.Gallileo was the first to show that the path of an object thrown in space is a parabola. Rope Bridges, artificial fountains are made by using the Properties of Parabola.

Ellipse:- If a point moves on a plane in such a way that its distance from a fixed point always bears a constant ratio to its perpendicular distance from a fixed line (not through fixed point) and if this ratio is less than 1 then the locus of that moving point is called an ellipse. The fixed Point is called focus and the fixed line is called the directrix and the constant ratio is called the eccentricity (e) of the ellipse.

Let S be the focus, L_1L_2 be the diretrix and e be the eccentricity of the ellipse. From the focus (S) perpendicu lar $\overline{ZS}$ drawn to the directrix. The line-segment $\overline{SZ}$ divided internally and enternally at A and A^1 respectively in the ratio e:1

$$\therefore \frac{\overline{SA}}{\overline{AZ}} = \frac{\overline{SA^1}}{\overline{AZ}} = e \quad \ldots \quad (1)$$

A and A^1 are called the vertices of the ellipse and the mid point of the line regment $\overline{AA^1}$ is called its Centre(C). AA^1 is called the major axis of the ellipse.

Let $\overline{AA^1} = 2a$.

$$\therefore \overline{CA} = \overline{CA^1} = a$$

Now from (1) we get, $\overline{SA} = e\overline{AZ}$ . . . (2)

and $\overline{SA^1} = e\overline{A^1Z}$ . . . (3)

from (2) and (3) we get,

$$\overline{SA} + \overline{SA^1} = e(\overline{AZ} + \overline{A^1Z})$$

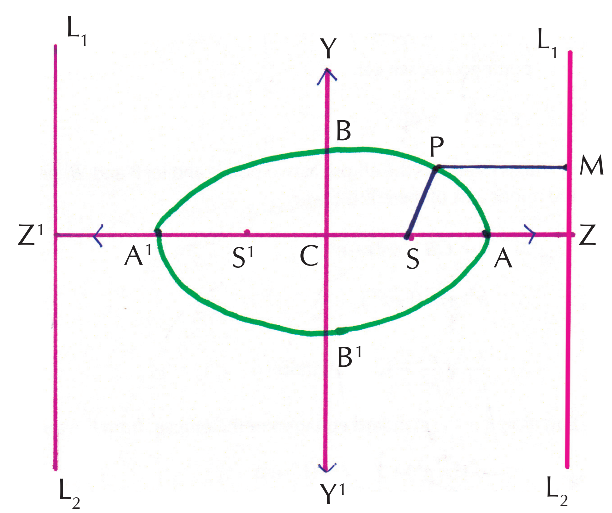
$$\overline{AA^1} = e[(\overline{CZ} - \overline{CA}) + (\overline{CZ} + \overline{CA^1})]$$

$$2a = e[(\overline{CZ} - a) + (\overline{CZ} + a)]$$

$$= 2e\,\overline{CZ}$$

$$\therefore a = e\,\overline{CZ}$$

$$\therefore \overline{CZ} = \frac{a}{e}$$



Again, Subtracting (2) from (3) we get

$$\overline{SA^1} - \overline{SA} = e(\overline{A^1Z} - \overline{AZ})$$

or, $(\overline{CS} + \overline{CA^1}) - (\overline{CA} + \overline{CS}) = C\overline{AA^1}$

or, $(\overline{CS} + a) - (a - \overline{CS}) = e.2a$

or, $2\overline{CS} = 2ea$

or, $\overline{CS} = ae$

Determination of the equation of ellipse:- Let S be the focus, $\overline{L_1L_2}$ be the directrix, C be the centre, $\overline{CZ}$ be the perpendicular to the directrix and A and A^1 be the vertices of the ellipse. So $\overleftrightarrow{AA'}$ is major axis of the ellipse. Let C be the centre, the straight line $\overleftrightarrow{CX}$ be taken as the X-axis and the Perpendicular drawn at C to the line $\overleftrightarrow{CX}$ is taken as Y-axis.also let the length of the major axis = 2a.

$\therefore \overline{CA} = \overline{CA'} = a$ and $\overline{CS} = ae$

So, the Co-ordinates of A, A^1 and the focus S are respectively (a,o) , (-a,o) and (ae,o). Now, if $\overleftrightarrow{CX}$ intersect the directix $\overleftrightarrow{L_1L_2}$ at the point Z then $\overline{CZ} = \frac{a}{e}$. Therefore the co-ordinate of Z be ($\frac{a}{e}$,o) and since two directrix L_1L_2 perpendicular to x-axis and passing through the point ($\frac{a}{e}$,o) then its equation is $x = \frac{a}{e}$.

Let P(x,y) be any point on the ellipse.

$\therefore$ from $\overline{Sp} = e\overline{PM}$ we get

$$\sqrt{(x-ae)^2 + y^2} = e(\frac{a}{e} - x)$$

or $$x^2(1-e^2) + y^2 = a^2(1-e^2)$$

or $$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1 \;..... (4)$$

putting x = o, we get

$$y = \pm a\sqrt{1-e^2}$$

$\therefore$ Y-axis intersects the ellipse at two points and let B and B^1 be that two points respectively and $\overline{BB'} = 2b$, $\overline{BB'}$ is the minor axis of the ellipse.

$\therefore$ b = $\overline{CB} = a\sqrt{1-e^2}$

$\therefore$ $b^2 = a^2(1-e^2)$

$\therefore$ $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 .... (5) \; (a>b)$$

Equation 5 is the standard equation of the ellipse. from the above equation we see that

$$x^2(1-e^2) + y^2 = a^2(1-e^2)$$

or, $$(x^2 - 2aex + a^2e^2) + y^2 = a^2 + 2aex + e^2x^2$$

or, $$(x^2 - 2aex + a^2e^2) + y^2 = a^2 + 2aex + e^2x^2$$

or, $$(x + ae)^2 + y^2 = (a + ex)^2$$

or, $$\frac{\sqrt{(x+ae)^2 + y^2}}{(x + \frac{a}{e})} = \pm e$$

∴ The numertor of the L.H.S. of the above equation is the distance between the points (-ae,o) and P(x,y) and denominator $(x+\frac{a}{e})$ is the distance of the line ex + a = o from the point P.

So taking the numerical value of e,it can say that the ratio of two distance from the point P(x,y) to the point (-ae,o) and the line ex + a = o is the eccentricity (e) of the ellipse. for this reason the second focus at the point (-ae,o) and the line ex + a = o i,e $x = -\frac{a}{e}$ is called the second directrix of the ellipse.

Two chords of the ellipse through its two foci and parallel to the directrix are called the latus recturn and it's length is $\frac{2b^2}{a}$.

If two axes of the ellipse are interchanged i,e x-axis as minor axis and y-axis as major axis then the equation of the ellipse is $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ (a>b)

Centre: C(o,o)

focii s, s^1 (o, ± ae)

Equation of dirctrix: $y = \pm\frac{a}{e}$

$(L_1L_2, L_1^1 L_2^1)$

Vertices $A, A^1 = (o, \pm a)$

$B, B^1 = (\pm b, o)$

$e = \sqrt{1-\frac{b^2}{a^2}}$

Properties of Ellipse:-

1. $\overline{SQ} = e\,\overline{SP}$ and the tangent and normal at any point P bisect the external and internal angles between the focal radio to the point.

2. If S^1T_1 and $S^1T_1^1$ are perpendicular from the foci upon the tangent at any point on the ellipse, then S^1T_1 x $S^1T_1^1 = b^2$ and T_1 & T_1^1 lies on the auniliary circle Also CT_1 and S^1P are parallel to each other.

3. If the normal at any point P on the ellipse meet the ,major axis and minor axis at Q and G respectively and if CF is Perpendicular to the normal then PF.PQ $= b^2$ and PF.PG $= a^2$

4. Parametric equation of the ellipse is

$x = a\cos\theta, y = b\sin\theta$, where θ is the eccentric angle.

Particular applications of Parabola :

In genneral use of ellipses can be found in physics, astronomy and engineering for example, the orbit of each planet in the solar system is elliptical. The same is true for moons orbiting planets the shapes of palnets and stars are often well described by ellipsoids.Ellipse can also arise as images of a circle under parallel projection and in same closed region one type of projection on plane such that it seems same objects are funthest from others. There are only intersections of general projections with the projections of right-circular cone.

Elliptical reflector and on which structure the expressability of extraordinaty sound is dependable.

In optics there is applicability of ellipse

Also there are many usage of ellipse in statistics and Economics.

Conclusions : We get a vast knowledge from theoritical discussion of parabola and ellipse and their practical applications.

Acknowledgement : Students will be grateful to those who have contributed to the implementation of this project.

References : 1. The Element of Co-ordinate Geometry (part-I) S.L. Loney

2. Geometry: B.C. Das & B.N. Mukherjee.

Project - 3

Title :- Concept of limit

Introduction :

Calculus is one of the most important part of Mathematics. Two of the most celebrated mathematicians of all time – Newton and Leibnitz, are considered to be the pioneer of this. Limit is a part of differentiation in calculus.

Objective :

The concept of limit absotutelly new in mathematics and is considered as the rudiment of calculus. Finding limit is a process by which one can evaluate the value of a function at the neighbourhood of a point in the domain of definition where the function is undefined. The use of the concept of limit in theoritical discussion of various branch of science and solving problems in economics is well-known to all.

Project procedure and data analysis :

According to the advice of subject teacher the subject matter that is the required concept, data and mathematical problems are collected from different books of calculus and from websites.

If we ask, what is the value of $\lim_{x\to 1}(2x+1)=?$, One can easily think that the value must be $2+1=3$. Then the question naturally arises – whether finding limit means putting the value of the variable in the given function but the answer is no. Because, if it is asked that what is the value of $\lim_{x\to 1}\frac{x^2-1}{x^2+1}=?$ then we don't write $\frac{1^2-1}{1^2+1}$. What we do $\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)}=\lim_{x\to 1}(x+1)=1+1=2$.

To understand what $x\to 1$ means, the following table is given

$x\to 1+0$	1.001	1.00001	1.0000001	$1.00000001\simeq 1$
$x\to 1-0$	.999	.99999	.9999999	$.999999999\simeq 1$

In the previous problem, as $x-1\neq 0$, so, $\lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)}=\lim_{x\to 1}(x+1)$

Again from the following table we can easily understand that the value of the function is 2.

$x\to 1+0$	$x+1$	2.001	2.00001	2.0000001	$2.00000001\simeq 2$
$x\to 1-0$	$x\to 1$	1.999	1.99999	1.999999	$1.999999999\simeq 2$

So, $\lim_{x\to a}f(x)$ exist iff $\lim_{x\to a+0}f(x)$ and $\lim_{x\to a-0}f(x)$ both exist and equal.

For example, if it is asked that what is the value of the limit $\lim_{x\to 0}\sin^{-1}\sec x=?$. Many can think of it is $\frac{\pi}{2}$. But the limit of the function does not exist as, when $x\to 0+0$, $\sec x>1$, but $\sin^{-1}x$ is defined for $-1\leq x\leq 1$. So, the function is undefined when $x\to 0+0$. So, the right hand limit does not exist and hence $\lim_{x\to 0}\sin^{-1}\sec x$ does not exist.

Few properties of limit :-

(i) $\lim_{x\to a}\{f(x)\pm g(x)\}=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$. It is notable that, $\lim_{x\to 1}\frac{x^2-1}{x-1}$ exist, but if we write $\lim_{x\to 1}\frac{x^2}{x-1}\pm\lim_{x\to 1}\frac{\mp 1}{x-1}$ then none of the above exist. So, it can be seen that – the existence of the limit of addition and subtraction of two function does not ensure the existence of the limit of the function individually. But if the individual limit of the function exist then the limit of addition and subtraction of the function will be defined :

(ii) $\lim_{x\to a}f(x)\,g(x)=\lim_{x\to a}f(x)\,\lim_{x\to a}g(x)$

Remark : $\lim_{x\to 0}x\sin\frac{1}{x}$ exit but $\lim_{x\to 0}\sin\frac{1}{x}$ doesn't exist.

So, it can be seen that – the existence of limit of two functions when they are multiplied doesn't ensure the existence of their individual limit but the converse is true in general.

(iii) $\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$ when $\lim f(x)$ and $\lim g(x)$ exist and $\lim g(x)\neq 0$.

(iv) $\lim_{x\to a}\{cf(x)\}=c\lim_{x\to a}f(x)$

(v) $\lim_{x\to a}\{f(x)\}^{g(x)}=\left\{\lim_{x\to a}f(x)\right\}^{\lim_{x\to a}g(x)}$ [when $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ are exist]

(vi) $\lim_{x\to a}f\{g(x)\}=f\{\lim_{x\to a}g(x)\}$

Analytical interpretation of the concept of limit :-

Let x be a real variable and *a* is a real constant also let f(x) be a single valued function of x.

Mathematically for $x\to a$, limit of f(x) is l iff for any arbitrarily small $\in>0,\ \exists\ \delta>0$ (depending on $\in$) such that $|f(x)-l|<\in$ whenever $0<|x-a|<\delta$.

Now, we know that, $|x-a|$ is the distance between x and a. So, we can state that if the distance between x and a minimizes then that of f(x) and f(a) also minimize.

For example, if $f(x)=x\sin\frac{1}{x}$ then $\left|x\sin\frac{1}{x}-0\right|=\left|x\sin\frac{1}{x}\right|=|x|\left|\sin\frac{1}{x}\right|$

$$\leq|x|\quad\left[\because\left|\sin\frac{1}{x}\right|\leq 1\right]$$

$$<\in$$

when $|x|<\varepsilon$ or $|x-0|<\delta$ (Let $\varepsilon=\delta$)

$\therefore\ \lim_{x\to a}x\sin\frac{1}{x}=0$

Remark : The above stated analytical concept of limit is also called ($\in - \delta$) definition of limit.

- **Finding limit of a function by ($\in - \delta$) definition** $\lim_{x\to 3}\frac{2x^2-18}{x-3}$.

Let, $f(x) = \lim_{x\to 3}\frac{2x^2-18}{x-3}$

Then, $\left|\frac{2x^2-18}{x-3}-12\right| = |2(x+3)-12| = 2|x-3|$

$\therefore |f(x)-12| < \varepsilon$, when $0 < |2(x-3)| < \varepsilon$

or, $0 < |x-3| < \frac{\varepsilon}{2} = \delta$

- **What is the meaning of f(x) $\to +\infty$ and $-\infty$ as x$\to$a?**

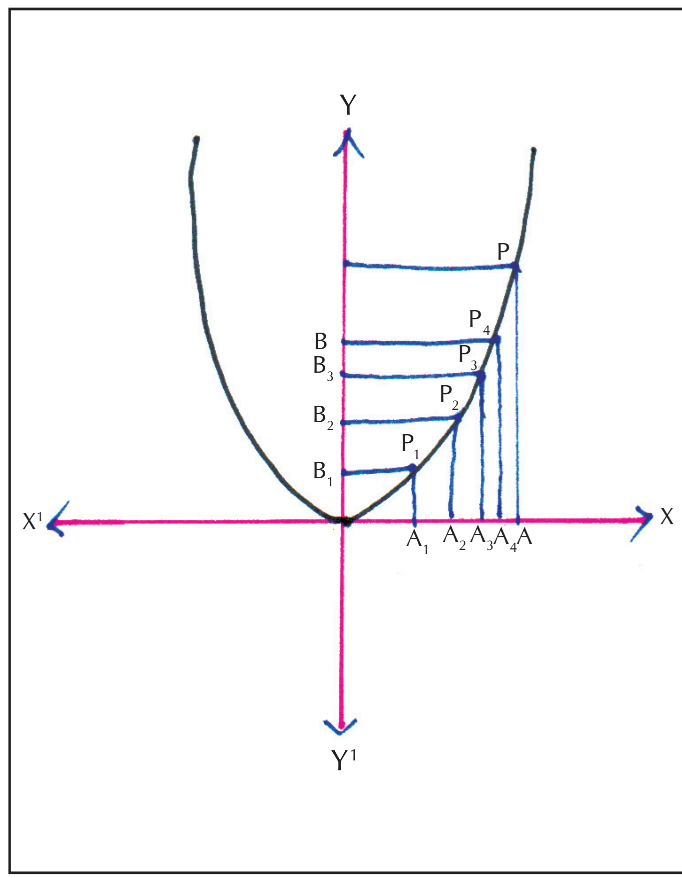
Let, x be a real variable, a be a real constant and f(x) is a single valued function of x. Then $f(x) \to +\infty$ as $x\to a$ means, as x approaches to a from right or from left in real line the functional value of f(x) gets bigger. If M is any large number then as $x\to a$, $f(x) > M$ and as $x\to a$, $f(x) \to -\infty$ means $f(x) < -M$.

Where M is any arbitrary large positive number.

For example :- $\lim_{x\to 1}\frac{1}{(x-1)^2} = +\infty$

Geometrical Concept of limit :-

Let, $f(x) = x^2$ be a continuous function. First the graph of f(x) is drawn.



Let, A(2, 0) be a point on the graph. $\overline{AP}$ is drawn parallel to Y-axis which cuts the parabola a point P(2, 4) and $\overline{PB}$ is drawn as parallel to X-axis if it intersect Y-axis at B(0, 4).

$\overline{OB} = \overline{AP} = 4$ is the value of the function when $x = 2$. Let a variable point takes the values $A_1, A_2 \ldots\ A_n$ consecutively on the left hand side of A along $\overline{OA}$.

Then $P_1, P_2, \ldots, P_n \ldots$ are the points on the graph of the function and $B_1, B_2, \ldots\ B_n, \ldots$ are the points.

On the left hand side of A gradually and Approches to A. (i.e. $x\to 2-0$) then the corresponding points on Y-axis will approach to B, that is $\lim_{x\to 2-0} x^2 = 4$.

Again, when the variable point takes the values A_1', A_2', ... A_n', ... from the right hand side of A along $\overline{OX}$ consecutively and $P_1', P_2', \ldots, P_n', \ldots$ are the corresponding points on graph of the function then we get similar corresponding points $B_1', B_2', \ldots, B_n', \ldots$ on Y-axis.

That is, if the variable point approaches to A from the right hand side (i.e., $x \to 2+0$) then correspondingly the similar point will approach to B on Y-axis i.e., $\lim_{x\to 2+0} x^2 = 4$ as $\lim_{x\to 2-0} x^2 = 4 = \lim_{x\to 2+0} x^2$.

So, the limiting value of f(x) exist and the limit is 4.

Conclusion :-

Theoretical study of limit will clear 'its' concept. As the concept of limit is the rudiment of calculus and in applied mathematics as well as in pure mathematics calculus is one of the most important tool, so, the theoretical concepts learned from this project will be helpful in future Education.

Acknowledge : Students will be grateful to those who have contributed to the implementation of this project.

Project - 4

Title : Nature of the curve of different function [eg. Algebric, Trigonometric, Greatest integer function, signum function etc.] and its tracing.

Introduction : There is a huge contribution of calculus in Modern Mathematics. The concept of calculus first introduced by Archimedes in third century B.C. Two scientists Newton (1642-1727) and Leibnitz (1646-1716) also plays an important role in the field of calculus. Latter on many Mathematicians and Scientists work in the field of calculus to develop this branch of mathematics. Many problems of mathematics and science are easily solved with the use of calculus. There are two branches of calculus,—

i) Integral Calculus

ii) Differential Calculus

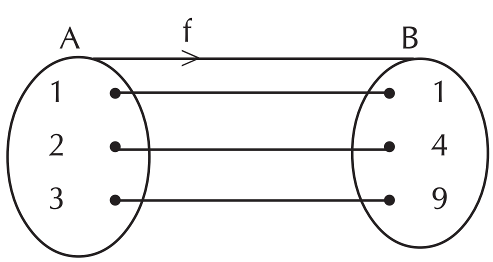
Function is a part of Differential Calculus.

Objectives : In a certain range, two variables are bound to one another with a certain rules. This introduce the concept of function in Math. To accure the indepth conception about the nature of curve of different function and its tracing is the main objective of this Project. From the nature of a function we can also know about the other Properties of a function. [i.e,– continuity or discontinuity, differentiable or not etc.]

Method & Analysis : Under the guidence of subject teacher necessary concept, information & problems are taken from different books of calculus & website.

If two non-empty sets are related to each other with a certain rule, then that particular rule is called Mapping. And it will be written as $f : A \to B$ [f maps A to B]

eg. If A & B are two non empty sets of real number and there is a relation between A & B as stated below.



i.e. $f(1) = 1$

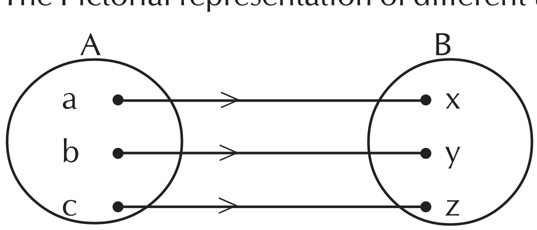
$f(2) = 4$

$f(3) = 9$ etc.

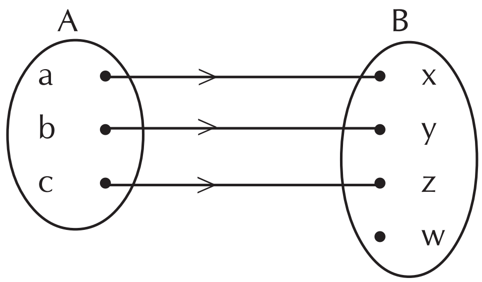
$\therefore$ It will be written as $f : A \to B$, where $f(x) = x^2$.

There are different types of mapping.

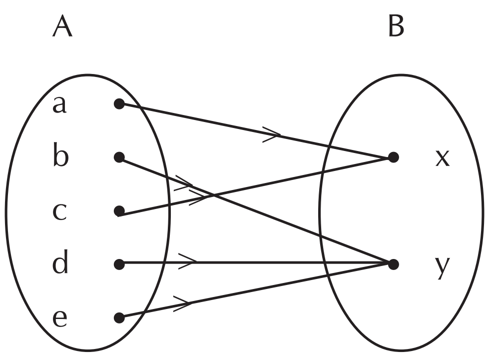
The Pictorial representation of different types of mapping are shown below :



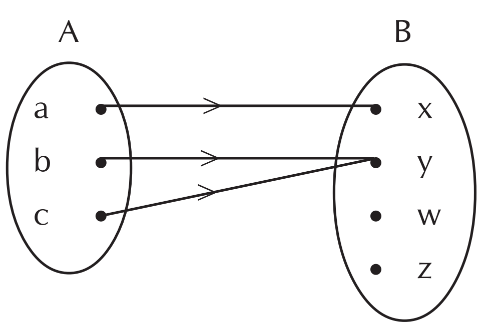
[One-one onto Mapping]



[One-one into Mapping]



[Many one onto Mapping]



[Many one into Mapping]

Definition of Function : Let x and y are two variables. If in a certain range, for each value of x there exist only one value of y, then y is called a single valued function of x or a function of x. The range of x will be the domain of the function and the range of y is the range of function.

eg : If $f(x) = \sqrt{x-1}$, then the domain of function is $\{x \in R, x \geq 1\}$ but the range of function is $[0, \infty)$.

Now, also we can get a function as $f : R \to R$, where $f(x) = \sin x$.

In this case domain is $-\infty < x < \infty$ or R.

But the range of f(x) in the domain is $[-1,1]$ which is a subset of R. Here R is called the co-domain of the function.

i.e, Range of function $\subseteq$ co-domain.

Classification of function

- Algebraic
  - Polynomial
  - Rational
  - Irrational
  - Piecewise
    - Modulus or Absolute valued function
    - Signum function
    - Greatest Integer function functions
    - Fractional Part function
    - Least Integer function
- Transcendental
  - Trigonometrical function
  - Exponential function
  - Logarithmic
  - Geometric curves
  - Inverse circular function

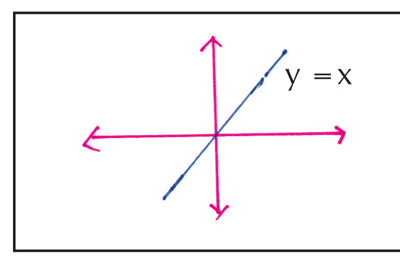
1 : Algebraic Function

a) Polynomial function :

It a function expressed as $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where $x \in N$ and $a_0, a_1, a_2, ..., a_n \in R$.

Then f is called Polynomial function.

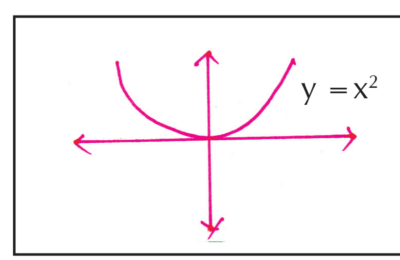
The Graphs or Tracing of some basic function are given below.



i) $f(x) = x$, for all $x \in R$

For drawing the graph x is independent variable, y is dependent variable and we have to find the value of x & y.

This function is called Identity function.



ii) $f(x) = x^2$

The domain of this function is R & Range is $[0, \infty)$

Here $f(-x) = (-x)^2$

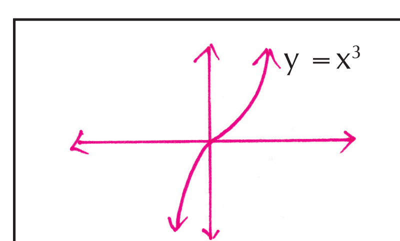
$= x^2$

$= f(x)$

Therefore this function is called even function. Also the curve of this function is symmetric with respect to y axis.

Therefore a function whose curve is symmetrical with respect to y axis is called even function.

This function is continuous & differentiable.



iii) $f(x) = x^3$

Both domain & Range of this function is set of real numbers.

As $f(-x) = (-x)^3$

$= -x^3$

$= -f(x)$

So, This function is an odd function.

From the curve of this function we can say that the curve on symmetric with respect to origin.

Also this function is continuous & differentiable.

b) Rational Function :

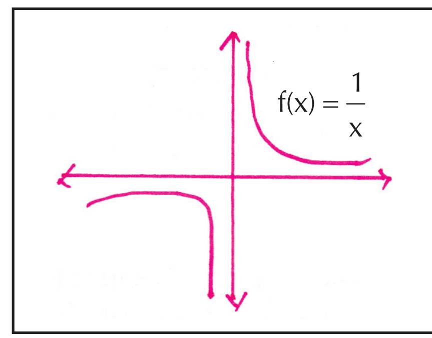
It $f(x) = \frac{p(x)}{q(x)}$ where p(x) & q(x) are Polynomial function.

Then f(x) is called Rational function.

Here, domain of function $\subset R - \{x \mid q(x) = 0\}$

The curve of some rational functions are shown below.

i) $f(x) = \frac{1}{x}$



The domain of this function is R–{0} and Range is $(-\infty, \infty)$.

This function is an odd function and at (0, 0) this function is discontinuous & not differentiable.

c) $f(x) = \frac{1}{x^2}$

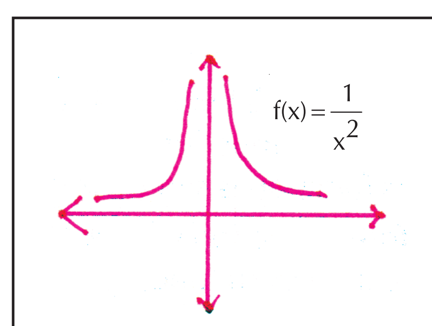
The domain of this function is R–{0} and Range is $(0, \infty)$

This is an even function and at (0, 0) this function is discontinuous & not differentiable.

c) Irrational Function :

The function where degree of variable is not a whole number ie, non integral rational Powers. Then the function is called Irrational function.

i) $f(x) = \frac{1}{x^2}$



Here, domain is $R^+ - \{0\}$

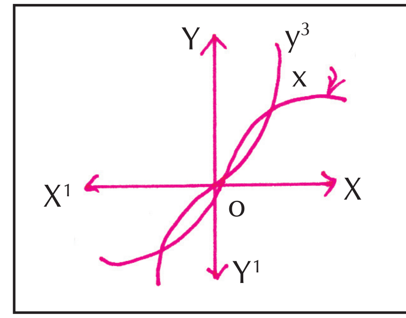
ie. if $f(x) = x^2$ & $g(x) = x^{1/2}$

them f(x) and g(x) are opposite functions of each other because both curves are mirror image with respect to y = x.

ii) **$f(x) = x^{1/3}$**

Here domain is R & Range is R.

Here $y = x^3$ and $y = x^{1/3}$ are inverse function of each other.



d) Modulus Function

f(x) = |x| i.e. $f(x) = x, x \geq 0$

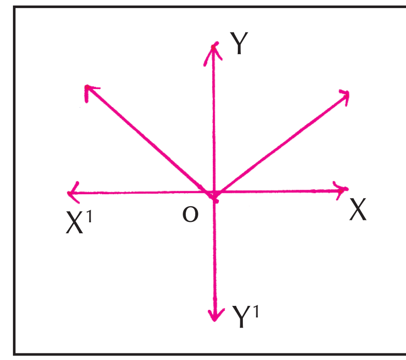
$= -x, x < 0$

It is an even function and its domain is R & range $[0, \infty)$

This function is continuous but at the point (0, 0) the curve makes a sharp bend.

So, this function is not differentiable at $x = 0$.

Now it can be said that if there in no intersect in the curve of any function, then function will be continuous everywhere and if the curve of a function makes a sharp bend at any point then the funtion is not differentiable.



Some Properties of this function is given below :

i) $|x| \leq a \Rightarrow -a \leq x \leq a \ (a \geq 0)$

ii) $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a \ (a \geq 0)$

iii) $|x+y| = |x| + |y| \Leftrightarrow x \geq 0, y \geq 0$
or, $x \leq 0, y \leq 0$

iv) $|x-y| = |x| - |y| \Leftrightarrow x \geq 0$ and $|x| \geq |y|$
and $y \leq 0$ and $|x| \geq |y|$

v) $|x \pm y| \leq |x| + |y|$

vi) $|x \pm y| \geq ||x| - |y||$

e) Signum Function :

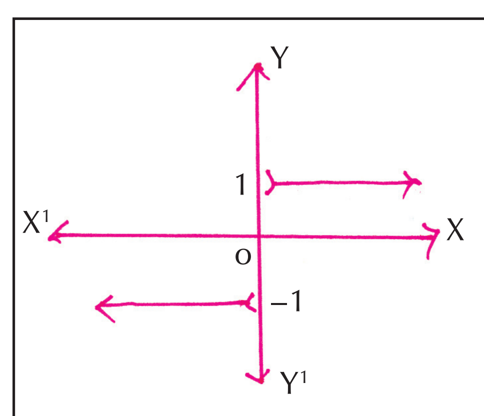
$f(x) = \text{sign}(x)$

Here domain is subset of R and range $\subset \{-1, 1\}$
$f(x) = \text{sign}(x)$

$= \frac{|x|}{x}; x \neq 0$

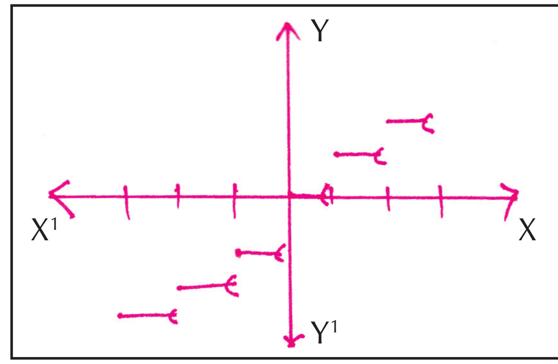
$0; \quad x = 0$

ie, $= +1$ when $x > 0$
$= -1$ when $x < 0$
$= 0$ when $x = 0$



From the nature of the curve of this function it can be said that it is an even function, continuous at (0, 0) and not differentiable.

f) Greatest Integer Function :



f(x) $= [x]$, where $[x]$ = Greatest Integer $\leqslant x$.

$= -2; -2 \leqslant x < -1$

$= -1, -1 \leqslant x < 0$

$= 0, \ 0 \leqslant x < 1$

$= 1, \ 1 \leqslant x < 2$

$= 2, \ 2 \leqslant x < 3$

The nature of the curve of this function is like steps. So, this function is called step function or Box function.

It is clear that the function is discontinuous and not differentiable.

The properties of this function are :

i) $[x] = x$, if x is whole number.

ii) $[x+I] = [x] + I$, if I is whole number

iii) $[x+y] \geqslant [x] + [y]$

iv) If $[\phi(x)] \leqslant I$, then $\phi(x) \geqslant I$

v) If $[\phi(x)] \leqslant I$, then $\phi(x) \leqslant I + 1$

vi) $[-x] = -[x]$, if x is whole number

vii) $[-x] = -[x] - I$, if x is not a whole number

viii) $x = [x] + \{x\}$, where $\{x\}$ is fraction of x.

2. Transcendental Function :

(a) Trigonometric Function

i) $f(x) = \sin(x)$

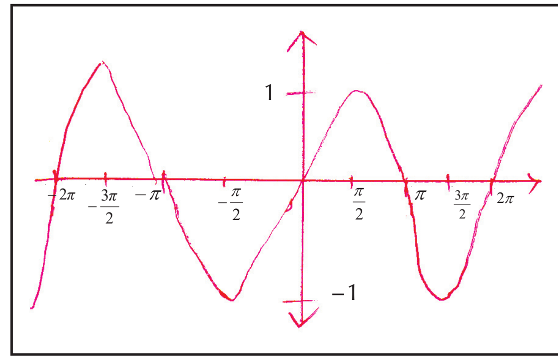
Here domain is R and range is $[-1, 1]$.

It is clear that the curve of the function is symmetric with respect to origin. So it is an odd function. Also we see that the curve of this function repeat at interval $[0, 2\pi]$. this function is called a Periodic Function.

Therefore the function f(x) is called the Periodic function of x if for every value of x, there exist a real number T, such that $f(x+T) = f(x)$. The smallest value of T is called the Period of the Periodic function.

eg. If $f(x) = \sin x$ then

$f(x+2\pi) = \sin(x + 2\pi) = \sin x = f(x)$



$f(x+4\pi) = \sin(x + 4\pi) = \sin x = f(x)$ etc.

It is seen that 2π is smallest, so the period of the function sin x is 2π.

ii) $f(x) = \cos x$

This function is even and Periodic function with Period 2π.

The domain is R and range is [–1, 1]

iii) $f(x) = \tan x$

It is an odd function. The domain is $R - \{(2n + 1)\frac{\pi}{2}, n \in I\}$ and range $\subset R$.

b) Inverse Circular Function :

A simple equation $\sin y = x$, where $|x| \leqslant 1$.......... (i)

Now (1) is satisfied for infinite value of y. Here the angle y is called $\sin^{-1}x$.

ie. $\sin^{-1}x$ is an angle but sinx is the ratio of side of a triangle.

Therefore if $f : [\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow [-1, 1]$, then the inverse function of $f(x) = \sin x$ will be

$y = \sin^{-1}x\ (-\frac{\pi}{2}\ x \leqslant \sin \frac{x}{2} \leqslant \frac{\pi}{2})$.

Similarly, we can found inverse function of $f(x) = \cos x$,

when $f : [0, \pi] \rightarrow [-1, 1]$.

We can find the inverse function of $f(x) = \tan x$, when $f : (-\frac{x}{2}, \frac{x}{2}) \rightarrow (-\infty, \infty)$

The inverse function of $y = \cot x$ when $f : (0, x] \rightarrow (-\infty, \infty)$.

The inverse function of $y = \sec x$ when $f : [0, \pi] - \{\frac{\pi}{2}\} \rightarrow R - (-1, 1)$.

The inverse function of $y = \text{cosec } x$ when $f : [-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\} \rightarrow R - (-1, 1)$.

Conclusion : Theoritical study of the nature of the curve will clear the concept of different type of function with its dommin of definition and range. So this project will be helpful in future education.

Acknowledgement : Students will be grateful to those who have contributed to the implementation of this project.

Some Suggested Project On Mathematics

1. History of Ancient number systems and its algorithms.
2. Use of graphs, different types of graphical representation, inferences about data from the graph.
3. Collection of statritical data and analysing it for standerd deviation and mean deviation.
4. Determination and comparision of areas bounded by a known cuvve (e.g. sinx, cos x, straight line, circle, parabola, ellipse) by definite integral and by dividing the total enclosed area into given number of small sub-intervals.
5. Detail discussion on Maxima and Minima of given function with special reference to the following cases.
   (i) Difference between local and global maxima and minima.
   (ii) Maxima and Minima at the points where the function is not differentiable with the terms like critical points, points of inflecion, stationary points, turning points and their iimplications.
6. Discusion on origin and formation of differential equations with reference to its application in on of the following cases :
   (i) Biological Problems
   (ii) Physical and Chemical Problems
   (iii) Geomatrical Problems
7. Discusion on Linear programming Problem related to Day to Day life like collecting data from families of their experditaresand requirements from factories to maximum outputs.
8. Discusion on different types of feasible and infeasible region for both bounded and unbounded region in linear Programming Problems.
9. History, development and application of matrix and matric algebra.
10. Project on the concept of inverse mapping (or function), condition of existence of inverse mapping composition of a function with its inverse function, inverse trigonometric function, interpretation of its Principal value and their graphical representation, verification of Properties of Inverse trigonomatric functions.

Notes

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